Day 184 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}-2\\-5\\0\end{pmatrix}$ and B has position vector $\begin{pmatrix}-6\\-3\\4\end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}-4\\2\\4\end{pmatrix}$

$\text{AP} = \frac{1}{2}\text{AB} = \begin{pmatrix}-2\\1\\2\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}-4\\-4\\2\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-4}\\\mathbf{-4}\\\mathbf{2}\end{pmatrix}$

2 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 13 pairs of data, $S_{xx} = 60$, $S_{yy} = 125$ and $S_{xy} = 67$.

Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{67}{\sqrt{60 \times 125}}$

$= \frac{67}{86.603} = 0.774$

r is positive, so there is strong positive correlation

Answer: r = 0.774, strong positive correlation

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist travelling at 14 m s^{-1} brakes and comes to rest with constant deceleration 2 m s^{-2}.

Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 14^2 - 2 \times 2 \times s$

$s = 196 \div 4 = 49$

(b) $0 = 14 - 2t$, so $t = 14 \div 2$

Answer: (a) 49 m (b) 7 s

4 Fixed-point iteration (All boards)

(a) Show that $e^x - x - 6 = 0$ can be written as $x = \ln(x + 6)$.

(b) Use $x_{n+1} = \ln(x_n + 6)$ with $x_0 = 1$ to find x_1, x_2, x_3 to 4 d.p.

(a) $e^x = x + 6$, so $x = \ln(x + 6)$

(b) $x_1 = 1.9459$

$x_2 = 2.0727$

$x_3 = 2.0885$

Answer: $x_1 = 1.9459$, $x_2 = 2.0727$, $x_3 = 2.0885$

5 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 1)(x + 2)(x - 5) > 0$.

Critical values: $x = -2$, $x = 1$, $x = 5$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $-2 < x < 1$ or $x > 5$

Answer: $-2 < x < 1$ or $x > 5$

6 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf f(x) = kx for $0 \le x \le 3$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 1). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^2/2]$ from 0 to 3 = $k \times \frac{3^2}{2} = 1$, so $k = \frac{2}{9}$

(b) $P(X > 1) = \int_1^3 \frac{2}{9} x\,dx = 1 - \frac{1^2}{3^2} = \frac{8}{9}$

(c) $E(X) = \int_0^3 x\,f(x)\,dx = \frac{2}{9} \times \frac{3^3}{3} = 2$

Answer: (a) $k = \frac{2}{9}$ (b) $\frac{8}{9}$ (c) 2

7 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 4 s.

cos α = 12/13, sin α = 5/13
R = mg cos α, F = μR
ma = mg sin α − μmg cos α, so a = 9.8(5/13 − 0.2 × 12/13)
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 4^2$

Answer: 1.96 m s^{-2}; 15.68 m

8 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0 e^{kt}$, where t is in hours. Initially N = 2000 and after 3 hours N = 4000.
Find (a) k to 3 s.f., (b) when N reaches 10000, to 3 s.f.

N_0 = 2000, so $4000 = 2000\,e^{3k}$, $e^{3k} = 2$

(a) $k = \frac{1}{3} \ln 2 = 0.2310$

(b) $2000\,e^{kt} = 10000$, so $t = \frac{\ln 5}{k} = 6.966$ hours

Answer: (a) k = 0.231 (b) t = 6.97 hours

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 2$.

Assume $x^2 - y^2 = 2$, so (x − y)(x + y) = 2.
x − y and x + y have the same parity (their sum 2x is even).
If both are odd, the product is odd; if both are even, the product is a multiple of 4.
But 2 is even and is not a multiple of 4 (2 = 4 × 0 + 2), a contradiction.
So no such integers exist.

Answer: no integer solutions (product is odd or a multiple of 4)

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 4x - 1$.

C is translated by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and then stretched by scale factor 2 parallel to the y-axis.

Find its new equation.

Translation: replace x by (x − h) and add k to y.

When translated by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$: $y = x^2 - 2x - 3$

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 2 parallel to the y-axis: $y = 2x^2 - 4x - 6$

Answer: $y = 2x^2 - 4x - 6$