Day 183 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Area under a curve (All boards)

The region R is bounded by the axes, the line $x = \ln 2$ and the curve $y = 2e^{2x} + 1$. Find the exact area of R.

Area $= \int_0^{\ln 2} (2e^{2x} + 1)\,dx = [x + e^{2x}]_0^{\ln 2}$

$= \ln 2 + 3$

Answer: $\ln 2 + 3$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $0 < -2x + 5 \leq 5$.

Subtract 5 from each part: $-5 < -2x \leq 0$

Divide each part by -2 (reversing the inequalities): $0 \leq x < \frac{5}{2}$

Answer: $0 \leq x < \frac{5}{2}$

3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{3}{2}$ and θ is acute, find the exact value of cosec θ.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan\theta = \frac{3}{2}$, so $\sec^2\theta = 1 + \frac{9}{4} = \frac{13}{4}$

$\cot\theta = \frac{2}{3}$, so $\operatorname{cosec}^2\theta = \frac{13}{9}$ and $\operatorname{cosec}\theta = \frac{\sqrt{13}}{3}$

Answer: $\frac{\sqrt{13}}{3}$

4 Inverse normal (All boards)

The mass X g of a flour bag is normal with mean 250, s.d. 12. Find k (3 s.f.): (a) $P(X < k) = 0.1$, (b) $P(X > k) = 0.01$.

(a) $z = -1.282$, so $k = 250 + (-1.282) \times 12 = 235$

(b) $P(X < k) = 0.99$, $z = 2.326$, so $k = 250 + 2.326 \times 12 = 278$

Answer: (a) 235 (b) 278

5 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 45 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 10 to t = 40. The floors are 675 m apart. Find V, and the deceleration in the final stage.

Area $= \frac{1}{2} \times (45 + 30) \times V = 675$

$75\,V = 1350$ so $V = 18$

Deceleration $= 18 \div 5$

Answer: V = 18; deceleration 3.6 m s^{-2}

6 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 4 per hour. Find the probability that in 15 minutes there are exactly 0 (3 s.f.).

Rate in 15 minutes: $\lambda = 4 \times \frac{15}{60} = 1$

$X \sim \text{Po}(1)$

$P(X = 0) = e^{-1} \times \frac{1^0}{0!} = 0.368$

Answer: 0.368

7 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{12}{5}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{5}{13}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{25}{39} T_2$; into the second: $T_2 \times \frac{56}{39} = 58.8$

Answer: $T_1 = 26.25$ N (at α); $T_2 = 40.95$ N (at β)

8 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 12 cm and the sides are folded up to make an open box.
Find x for the greatest volume, and that volume.

$V = x(12 - 2x)^2 = 4x^3 - 48x^2 + 144x$

$\frac{dV}{dx} = 12x^2 - 96x + 144 = (6x - 12)(2x - 12) = 0$

$x < 6$, so $x = 2$

$\frac{d^2V}{dx^2} = 24x - 96 = -48 < 0$ at $x = 2$, so maximum

$V = 2 \times 8^2 = 128$

Answer: $x = 2$, $V = 128$ cm^3

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x + 4y = k$ is a tangent to the circle
$(x + 3)^2 + (y - 4)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
$(-3, 4) \pm 5 \times (3, 4)/5 = (-3, 4) \pm (3, 4)$
T = (0, 8) or (−6, 0), and $k = 3x + 4y$ gives:
k = 32 or k = −18
Answer: k = 32 or k = −18

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{5}$ is irrational.

Assume $\sqrt{5} = \frac{a}{b}$, where a and b are integers with no common factor.

Then $a^2 = 5b^2$, so a^2 is a multiple of 5, so a is a multiple of 5.
Let $a = 5k$. Then $5^2k^2 = 5b^2$, so $b^2 = 5k^2$.
So b^2, and hence b, is a multiple of 5.
Then a and b have a common factor 5, a contradiction, so $\sqrt{5}$ is irrational.
Answer: $\sqrt{5}$ is irrational