Day 182 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{4}} (4 \sin 2x + 5 \cos 2x)\, dx$.

$\int (4 \sin 2x + 5 \cos 2x)\, dx = \frac{5 \sin 2x}{2} - 2 \cos 2x$

$= [\frac{5 \sin 2x}{2} - 2 \cos 2x]_0^{\frac{\pi}{4}}$

$= \frac{9}{2}$

Answer: $\mathbf{\frac{9}{2}}$

2 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 5 kg is moving on a smooth horizontal surface at 8 m s^{-1} when a constant horizontal force of 6 N, in the direction of motion, acts for 2 s.

Find the speed of the trolley at the end of this time and the distance it moves.

F = ma: a = 6 ÷ 5 = 1.2

v = u + at = 8 + 1.2 × 2

s = ut + ½at^2

Answer: 10.4 m s^{-1}; 18.4 m

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 + 8x \leq 15$.

Rearrange: $-x^2 + 8x - 15 \leq 0$

Factorise: $-(x - 5)(x - 3) \leq 0$

Critical values: x = 3 and x = 5

Sketch the ∩-shaped parabola: $x \leq 3$ or $x \geq 5$

Answer: $x \leq 3$ or $x \geq 5$

4 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 2 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan \alpha = \frac{5}{12}$.

Take g = 9.8 m s^{-2}.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

cos α = 12/13, sin α = 5/13

(a) Horizontal: 39 × 12/13 = 2 a

(b) Vertical: R + 39 × 5/13 = 2 × 9.8

Answer: (a) 18 m s^{-2} (b) 4.6 N

5 Laws of logarithms: solving equations (All boards)

Solve 2 ln x = ln(x + 12).

ln x^2 = ln(x + 12), so $x^2 = x + 12$

$x^2 - x - 12 = 0$, so $(x - 4)(x + 3) = 0$

x > 0 for ln x to exist, so x = 4

Answer: x = 4

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6 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t - 2$, $y = t^2 - 4t - 3$.
The line $y = -x - 3$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 - 4t - 3 = -3t - 1$
$t^2 - t - 2 = 0$, so $(t - 2)(t + 1) = 0$
$t = -1$ or $t = 2$, giving the points $(-5, 2)$ and $(4, -7)$
Answer: $(-5, 2)$ and $(4, -7)$

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{x + 6}{(x + 2)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{x + 6}{(x + 2)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{x + 6}{(x + 2)(x + 4)} \equiv \frac{A}{x + 2} + \frac{B}{x + 4}$, so $x + 6 \equiv A(x + 4) + B(x + 2)$
$x = -2$: $A = 2$; $x = -4$: $B = -1$
(b) $\int_0^4 \left(\frac{2}{x + 2} + \frac{-1}{x + 4}\right) dx = [2\ln(x + 2) - \ln(x + 4)]_0^4$
$= 2\ln\frac{6}{2} - \ln\frac{8}{4} = \ln\left(\frac{9}{2}\right)$

Answer: (a) $A = 2$, $B = -1$ (b) $\ln\left(\frac{9}{2}\right)$

8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.2. It is tossed 30 times and lands heads 13 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.2.

H_0: $p = 0.2$, H_1: $p \neq 0.2$. Under H_0, $X \sim B(30, 0.2)$
$P(X \geq 13) = 1 - P(X \leq 12) = 0.0031$
Two-tailed: compare with 0.025. $0.0031 < 0.025$, so reject H_0
There is sufficient evidence that the probability of heads is different from 0.2
Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.2

9 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A salary starts at £24000 in year 1 and increases by 3% each year.
Find the total salary earned in the first 10 years, to the nearest pound.

Geometric series with $a = 24000$, $r = 1.03$
$S_{10} = \frac{24000(1.03^{10} - 1)}{0.03}$
= £275,133
Answer: £275,133

10 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(100, 0.4)$. Use a normal approximation to find $P(X \geq 44)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(40.00, 24.00)$, so $\sigma = \sqrt{24.00} = 4.899$
Use a continuity correction.
$P(X \geq 44) \approx P(Y > 43.5) = P(Z > 0.7144) = 0.237$
Answer: 0.237