Day 181 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 70 kg stands on the floor of a lift. The lift accelerates upwards at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 70 \times 9.8 = 70 \times 2$
$R = 70(9.8 + 2)$
Answer: 826 N

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^3)^2 \times x}{x^5}$.

$(2x^3)^2 = 4x^6$
Numerator = $4x^6 \times x = 4x^7$
Divide: $\frac{4}{1} = 4$ and $x^7 \div x^5 = x^2$
Answer: $4x^2$

3 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}-3\\-3\\1\end{pmatrix}$, $\begin{pmatrix}-6\\-5\\0\end{pmatrix}$ and $\begin{pmatrix}p\\-7\\-1\end{pmatrix}$ respectively.
Given that A, B, C are collinear, find p.

$AB = \begin{pmatrix}-3\\-2\\-1\end{pmatrix}$, $AC = \begin{pmatrix}p - -3\\-4\\-2\end{pmatrix}$
$AC = \lambda AB$: component 2 gives $-4 = \lambda \times -2$, so $\lambda = 2$
$p - -3 = 2 \times -3$, so $p = -9$
Answer: $p = -9$

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Boys: $65 - 40 = 25$. Girls: $40 - 30 = 10$
(b) Medians: 50 (Boys) and 35 (Girls), so Boys has the higher median
IQRs: Boys has the larger IQR, so its scores are more spread out
Answer: (a) 25 and 10 (b) Boys has the higher median; Boys is more spread out (larger IQR), Girls more consistent

5 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{2}} (2x - 1)\cos x \, dx$.

Let $u = 2x - 1$ and $\frac{dv}{dx} = \cos x$, so $\frac{du}{dx} = 2$ and $v = \sin x$
Integral = $[uv] - \int v\frac{du}{dx} \, dx$
$= [(2x - 1)\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\sin x \, dx$
$= \pi - 3$
Answer: $\pi - 3$

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6 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 11 pairs of data, $S_{xx} = 200$, $S_{yy} = 100$ and $S_{xy} = -127$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-127}{\sqrt{200 \times 100}}$

$= \frac{-127}{141.42} = -0.898$

r is negative, so there is strong negative correlation

Answer: r = −0.898, strong negative correlation

7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan^2 x - 5 \sec x + 7 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $\sec^2 x - 5 \sec x + 6 = 0$
$(\sec x - 3)(\sec x - 2) = 0$, so $\sec x = 3$ or 2
$\sec x = 3$, so $\cos x = \frac{1}{3}$: $x = 70.5°, 289.5°$
$\sec x = 2$, so $\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

Answer: x = 60.0°, 70.5°, 289.5°, 300.0°

8 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 6| = |x|$.

Either $2x + 6 = x$ or $2x + 6 = -(x)$
$2x + 6 = x$ gives $x = -6$
$2x + 6 = -(x)$ gives $x = -2$

Answer: x = −6 and x = −2

9 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 9 kg and 11 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take $g = 9.8$ m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: $11g - T = 11a$
Lighter: $T - 9g = 9a$
Add: $2g = 20a$, so $a = 0.98$
$T = 9(9.8 + 0.98)$

Answer: a = 0.98 m s^{-2}; T = 97.02 N

10 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:

(a) $y = f(\frac{x}{2})$ (b) $y = f(x - 5)$

(a) $y = f(\frac{x}{2})$: A(1, −4) maps to (2, −4), B(5, 5) maps to (10, 5)

(b) $y = f(x - 5)$: A(1, −4) maps to (6, −4), B(5, 5) maps to (10, 5)

Answer: (a) (2, −4), (10, 5) (b) (6, −4), (10, 5)