Day 180 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 42.35 cm^2 and angle 0.7 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 42.35$, so $r^2 = \frac{2 \times 42.35}{0.7} = 121$, $r = 11$

Perimeter $= 2r + r\theta = 22 + 7.7 = 29.7$ cm

Answer: (a) 11 cm (b) 29.7 cm

2 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (5, 2) and passes through the point (−1, 6).
Find an equation of the circle.

$r^2 = (-1 - 5)^2 + (6 - 2)^2$
$= 36 + 16 = 52$

Answer: $(x - 5)^2 + (y - 2)^2 = 52$

3 Probability without replacement (All boards)

A bag has 3 red and 9 blue counters; two are taken without replacement.
(a) Find P(same colour).
(b) Given they are the same colour, find P(both red).

(a) P(red red) $= \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

P(blue blue) $= \frac{9}{12} \times \frac{8}{11} = \frac{6}{11}$

P(same) $= \frac{1}{22} + \frac{6}{11} = \frac{13}{22}$

(b) P $= \frac{1}{22} \div \frac{13}{22} = \frac{1}{13}$

Answer: (a) $\frac{13}{22}$ (b) $\frac{1}{13}$

4 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 4x + 10y + 13 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x - 2)^2 - 4 + (y + 5)^2 - 25 + 13 = 0$
$(x - 2)^2 + (y + 5)^2 = 16$
Centre (2, −5), radius 4

Answer: centre (2, −5), radius 4

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan\alpha = \frac{12}{5}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in the string and the value of P.

$\cos\alpha = 5/13$, $\sin\alpha = 12/13$
Vertical: $T\cos\alpha = mg = 58.8$, so $T = 58.8 \times 13/5$
Horizontal: $P = T\sin\alpha = 152.88 \times 12/13$

Answer: T = 152.88 N; P = 141.12 N

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6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{6y}{x}$ for $x > 0$,
given that $y = 2$ when $x = 1$.

$\frac{1}{y}\,dy = \frac{6}{x}\,dx$, so $\ln y = 6 \ln x + c$
$x = 1, y = 2$: $c = \ln 2$
$y = 2x^6$
Answer: $y = 2x^6$

7 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg lies on a rough horizontal table (coefficient of friction 0.1). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.1 \times 5g = 5a$
Hanging: $5g - T = 5a$
Add: $5g - 0.5g = 10a$
$a = 4.41$
$T = 5(9.8 - 4.41)$
Answer: $a = 4.41$ m s^{-2}; $T = 26.95$ N

8 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.1, find the least number of components needed for the probability of at least one success to exceed 0.9.

P(at least one) $= 1 - 0.9^n > 0.9$
$0.9^n < 0.1$
$n > \frac{\log 0.1}{\log 0.9} = 21.85$
Answer: $n = 22$

9 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $2\sin^2 x + 3\sin x + 1 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(\sin x + 1)(2\sin x + 1) = 0$, so $\sin x = -1$ or $\sin x = -\frac{1}{2}$
$\sin x = -1$: $x = 270.0°$
$\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$
Answer: $x = 210.0°, 270.0°, 330.0°$

10 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(8, −4) and B(−7, 1), and its centre lies on the x-axis. Find the centre and the equation of the circle.

Let the centre be (h, 0). Then $CA^2 = CB^2$:
$(h - 8)^2 + 16 = (h + 7)^2 + 1$
Expanding and cancelling h^2: $-16h + 80 = 14h + 50$
$h = 1$
$r^2 = CA^2 = 65$
Answer: centre (1, 0), $(x - 1)^2 + y^2 = 65$