Day 18 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 11 students:
12, 12, 16, 18, 18, 21, 25, 28, 28, 30, 34.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 16$, $Q_3 = 28$, IQR = 12
Lower limit = $16 - 1.5 \times 12 = -2$
Upper limit = $28 + 1.5 \times 12 = 46$
No value lies outside these limits.
Answer: $Q_1 = 16$, $Q_3 = 28$, IQR = 12; no outliers

2 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(9 - x)^{-\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 9: $(9 - x)^{-\frac{3}{2}} = 9^{-\frac{3}{2}}(1 + -\frac{1}{9}x)^{-\frac{3}{2}} = \frac{1}{27}(1 + -\frac{1}{9}x)^{-\frac{3}{2}}$

Expand $(1 + -\frac{1}{9}x)^{-\frac{3}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by $\frac{1}{27}$: coefficients of 1, x, x^2, x^3 are $\frac{1}{27}$, $\frac{1}{162}$, $\frac{5}{5832}$, $\frac{35}{314928}$

Valid when $|\frac{1}{9}x| < 1$, that is $|x| < 9$

Answer: (a) $\frac{1}{27} + \frac{1}{162}x + \frac{5}{5832}x^2 + \frac{35}{314928}x^3 + ...$ (b) $|x| < 9$

3 Trig equations in radians (All boards)

Solve $\tan x = -3$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

$\tan \theta = -3$ for $0 \leq x \leq 2\pi$
Calculator: $x = -1.249$, then use the symmetry of the graph
$x = 1.893, 5.034$
Answer: x = 1.89, 5.03

4 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = −f(x)
(b) y = f(3x)

(a) y = −f(x) is a reflection in the x-axis.

(b) y = f(3x) is a stretch, scale factor $\frac{1}{3}$, parallel to the x-axis.

Answer: (a) reflection in the x-axis (b) stretch, scale factor $\frac{1}{3}$, parallel to the x-axis

5 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-2x - 9}{(x + 4)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-2x - 9}{(x + 4)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-2x - 9}{(x + 4)(x + 5)} \equiv \frac{A}{x + 4} + \frac{B}{x + 5}$, so $-2x - 9 \equiv A(x + 5) + B(x + 4)$
$x = -4$: $A = -1$; $x = -5$: $B = -1$

(b) $\int_0^4 (\frac{-1}{x + 4} + \frac{-1}{x + 5})\,dx = [-\ln(x + 4) - \ln(x + 5)]_0^4$
$= -\ln \frac{8}{4} - \ln \frac{9}{5} = \ln(\frac{5}{18})$

Answer: (a) A = −1, B = −1 (b) $\ln(\frac{5}{18})$

6 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 4 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 1 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(5) = 4\,g(2.5) + 1\,g(1.5)$

$T_2 = 22.54$

$T_1 + T_2 = 5\,g$ so $T_1 = 49 - 22.54$

Answer: Near A: 26.46 N; near B: 22.54 N

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 200 g; 50 packets have mean 201.8 g.
Test at the 5% level whether the true mean is greater than 200.

$H_0: \mu = 200$, $H_1: \mu > 200$. Under H_0, $\overline{X} \sim N(200, \frac{5^2}{50})$

$z = \frac{201.8 - 200}{5/\sqrt{50}} = 2.546$

$P(Z > 2.546) = 0.0055 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass M kg of a substance satisfies $\frac{dM}{dt} = -0.15M$, where t is in years.

Initially M = 60. Find M when t = 10, and the half-life, each to 3 s.f.

$\frac{1}{M}\,dM = -0.15\,dt$, so $\ln M = -0.15t + c$, giving $M = 60e^{-0.15t}$

t = 10: $M = 60\,e^{-1.5} = 13.39$

Half-life: $e^{-0.15t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.15} = 4.621$

Answer: M = 13.4 kg, half-life 4.62 years

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 1 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 + 0y - 1 = 0$

$(y - 1)(y + 1) = 0$

$y = 1$ or $y = -1$

$2^x > 0$ for all x, so $y = -1$ is rejected. $2^x = 1$ gives $x = 0$.

Answer: x = 0

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{3}{4}$.

Take g = 9.8 m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{12}{13} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{13}{15}\,T_2$; into the second: $T_2 \times \frac{14}{15} = 58.8$

Answer: T_1 = 54.6 N (at α); T_2 = 63 N (at β)