Day 179 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 40 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 40$, $\Sigma fx = 85$, $\Sigma fx^2 = 271$

Mean = $\frac{85}{40} = 2.12$

Variance = $\frac{271}{40} - (2.1250)^2 = 2.259$

Standard deviation = 1.50

Answer: mean 2.12, standard deviation 1.50

2 Trig equations in radians (All boards)

Solve $\cos 2x = -0.6$ for $0 \leq x \leq \pi$,
giving answers to 3 significant figures.

Let $\theta = 2x$, so $\cos \theta = -0.6$ for $0 \leq \theta \leq 2\pi$
Calculator: $\theta = 2.214$, then use the symmetry of the graph
$\theta = 2.214, 4.069$

$x = \frac{\theta}{2}$

$x = 1.107, 2.034$

Answer: x = 1.11, 2.03

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 12 kg rests on a rough horizontal floor, with coefficient of friction 0.3. A horizontal force of 42 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 12 \times 9.8 = 117.6$, maximum friction $\mu R = 0.3 \times 117.6 = 35.28$
$42 > 35.28$ so the box moves, $F = 35.28$
$42 - 35.28 = 12\,a$

Answer: It moves; a = 0.56 m s^{-2}

4 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ 1 \\ 6 \end{pmatrix}$.

Find AB and the distance AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} 12 \\ -3 \\ 4 \end{pmatrix}$

$|\mathrm{AB}| = \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} = 13$

Answer: AB = $\begin{pmatrix} \mathbf{12} \\ \mathbf{-3} \\ \mathbf{4} \end{pmatrix}$, distance 13

5 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-6, 2)$ and $(3, -3)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-6+3}{2}, \frac{2+(-3)}{2}\right) = \left(-\frac{3}{2}, -\frac{1}{2}\right)$

(b) Gradient $= \frac{-5}{9} = -\frac{5}{9}$

$y - 2 = -\frac{5}{9}(x + 6)$

Multiply out and collect terms: $5x + 9y + 12 = 0$

Answer: (a) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (b) $5x + 9y + 12 = 0$

6 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $3t^2 - 18t + 24$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $3(t - 2)(t - 4) = 0$, so $t = 2$ or $t = 4$
First time $t = 2$

$s = \int_0^2 v\,dt = [\,t^3 - 9t^2 + 24t\,]$ from 0 to 2

Answer: t = 2 s; 20 m

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 10 ml. Set mean 50 ml; 16 bottles have mean 52.6 ml.
Test at the 5% level whether the true mean differs from 50.

$H_0: \mu = 50$, $H_1: \mu \neq 50$. Under H_0, $\overline{X} \sim N\left(50, \frac{10^2}{16}\right)$

$z = \frac{52.6 - 50}{10/\sqrt{16}} = 1.040$

$P(Z > 1.040) = 0.1492 > 0.025$
Do not reject H_0: there is insufficient evidence that the mean has changed

Answer: do not reject H_0: insufficient evidence of a change in the mean

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x + 4y = k$ is a tangent to the circle
$x^2 + y^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
$(0, 0) \pm 5 \times (3, 4)/5 = (0, 0) \pm (3, 4)$
T = (3, 4) or (−3, −4), and $k = 3x + 4y$ gives:
$k = 25$ or $k = -25$

Answer: k = 25 or k = −25

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9 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 24 cm and the sides are folded up to make an open box.

Find x for the greatest volume, and that volume.

$V = x(24 - 2x)^2 = 4x^3 - 96x^2 + 576x$

$\frac{dV}{dx} = 12x^2 - 192x + 576 = (6x - 24)(2x - 24) = 0$

$x < 12$, so $x = 4$

$\frac{d^2V}{dx^2} = 24x - 192 = -96 < 0$ at $x = 4$, so maximum

$V = 4 \times 16^2 = 1024$

Answer: x = 4, V = 1024 cm^3

10 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 5 and common ratio 2.

Find the smallest value of n for which the sum of the first n terms exceeds 500.

$S_n = 5(2^n - 1) > 500$

$2^n > 101$

$n > \frac{\lg 101}{\lg 2} = 6.658$

So the smallest integer n is 7

Answer: n = 7