Day 178 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is $(-6, 4)$ and B is $(-8, 2)$.
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = $(-7, 3)$
Radius^2 = $(-6 - (-7))^2 + (4 - 3)^2 = 2$
Answer: $(x + 7)^2 + (y - 3)^2 = 2$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 12 cm and arc length 15.6 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{15.6}{12} = 1.3$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times 1.3 = 93.6$

Answer: (a) 1.3 rad (b) 93.6 cm^2

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 8 m s^{-1} brakes and comes to rest with constant deceleration 4 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 8^2 - 2 \times 4 \times s$
$s = 64 \div 8 = 8$
(b) $0 = 8 - 4t$, so $t = 8 \div 4$
Answer: (a) 8 m (b) 2 s

4 Normal distribution probabilities (All boards)

A battery lasts X hours, normal with mean 170, s.d. 6.
Find, to 3 s.f., the probability a value is
(a) less than 176,
(b) between 176 and 182.

$X \sim N(170, 6^2)$
(a) $P(X < 176) = 0.841$
(b) $P(176 < X < 182) = 0.9772 - 0.8413 = 0.136$
Answer: (a) 0.841 (b) 0.136

5 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 2)^2 + (y + 1)^2 = 13$.
The point P(0, 2) lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = $(-2, -1)$. Gradient of radius CP = $\frac{3}{2}$

The tangent is perpendicular to the radius: gradient = $-\frac{2}{3}$

$y - 2 = -\frac{2}{3}x$

Multiply out and collect terms: $2x + 3y - 6 = 0$
Answer: $2x + 3y - 6 = 0$

6 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A missile is projected horizontally at 392 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 42 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 42 = 411.6
Speed = $\sqrt{(392^2 + 411.6^2)}$ = 568.4
tan α = 411.6 ÷ 392, α = 46.4°
Answer: 568.4 m s^{-1} at 46.4° below horizontal

7 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = ln x + x − 6. Use the Newton-Raphson method with
$x_0 = 5$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 5 - \frac{0.6094}{1.200} = 4.4921$

$x_2 = 4.4921 - \frac{-0.005537}{1.223} = 4.4967$

Answer: x_1 = 4.4921, x_2 = 4.4967

8 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Year 12: 40 − 25 = 15. Year 13: 55 − 35 = 20
(b) Medians: 35 (Year 12) and 50 (Year 13), so Year 13 has the higher median
IQRs: Year 13 has the larger IQR, so its scores are more spread out
Answer: (a) 15 and 20 (b) Year 13 has the higher median; Year 13 is more spread out (larger IQR), Year 12 more consisten

9 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = \frac{1}{2}$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{40} and $\sum_{r=1}^{40} u_r$.

(a) $u_2 = 2$, $u_3 = -1$, $u_4 = \frac{1}{2}$
$u_4 = u_1$ so the sequence repeats with period 3
(b) 40 = 3 × 13 + 1, so $u_{40} = u_1 = \frac{1}{2}$
Sum of one cycle = $\frac{3}{2}$, so the sum = $13 \times \frac{3}{2} + \frac{1}{2} = 20$
Answer: period 3; $u_{40} = \frac{1}{2}$; sum = 20

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10 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{4}} (\sin^2 2x + \cos 4x)\, dx$.

Use $\sin^2 2x = \frac{1}{2}(1 - \cos 4x)$

$\int (\sin^2 2x + \cos 4x)\, dx = \frac{x}{2} + \frac{\sin 4x}{8}$

$= \left[\frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8}$

Answer: $\boldsymbol{\frac{\pi}{8}}$