Day 177 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 7 \\ -6 \\ 6 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{7^2 + 6^2 + 6^2} = \sqrt{121} = 11$

Unit vector $= \frac{1}{11}\mathbf{a}$

Answer: $|\mathbf{a}| = 11$, unit vector $\frac{1}{11}\begin{pmatrix} 7 \\ -6 \\ 6 \end{pmatrix}$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-3x - 1 > 3x - 1$.

$-6x > 0$

Divide by -6 (reversing the inequality): $x < 0$

Answer: $x < 0$

3 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg moves along a straight horizontal road. The engine produces a driving force of 1800 N and the resistance to motion is 900 N.

Find the acceleration of the car.

Resultant force $= 1800 - 900 = 900$ N

$F = ma$: $900 = 1500\,a$

Answer: 0.6 m s^{-2}

4 Inequalities with exponents and logs (All boards)

Solve $2^x > 52$, giving your answer to 3 s.f.

Take logs: $x \ln 2 > \ln 52$

$x > \frac{\ln 52}{\ln 2} = 5.700$

Answer: $x > 5.70$

5 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + kx^2 + 19x + 12$.

Given that $(x + 1)$ is a factor of $f(x)$,

(a) find the value of k,

(b) factorise $f(x)$ completely.

$f(-1) = 0$: $-1 + k - 19 + 12 = 0$

$k = 8$

$f(x) = (x + 1)(x^2 + 7x + 12)$

$= (x + 1)(x + 3)(x + 4)$

Answer: (a) $k = 8$ (b) $f(x) = (x + 1)(x + 3)(x + 4)$

6 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 5 m and mass 80 kg rests horizontally on supports at C and D, where AC = 1.5 m and DB = 2 m.

A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $80\,g \times (3 - 2.5) = 20\,g \times x$

$80 \times 0.5 = 20\,x$

Answer: 2 m

Day 177 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 3 consecutive odd integers is always a multiple of 3.

Let the integers be 2n + 1, 2n + 3, 2n + 5
Sum = 6n + 9
= 3(2n + 3)
n is an integer, so (2n + 3) is an integer, so the sum is a multiple of 3.
Answer: sum = 3(2n + 3), a multiple of 3

8 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.5. In a sample of 50 students, 31 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.5.

H_0: p = 0.5, H_1: p ≠ 0.5. Under H_0, X ~ B(50, 0.5)
P(X ≥ 31) = 1 − P(X ≤ 30) = 0.0595
Two-tailed: compare with 0.025. 0.0595 > 0.025, so do not reject H_0
There is insufficient evidence that the probability is different from 0.5
Answer: do not reject H_0: insufficient evidence that the probability is different from 0.5

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 9 cos θ + 12 sin θ ≡ 15 cos(θ − 53.1°),
solve 9 cos θ + 12 sin θ = −0.5 for 0 ≤ θ ≤ 360°. (1 d.p.)

$15\cos(\theta - 53.1°) = -0.5$, so $\cos(\theta - 53.1°) = \frac{-0.5}{15}$
θ − 53.1° = ±91.9°
θ = 53.1° + 91.9° = 145.0°
θ = 53.1° − 91.9° = −38.8°, so θ = 321.2°
Answer: θ = 145.0°, 321.2°

10 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.3, find the least number of traps needed for the probability of at least one success to exceed 0.99.

P(at least one) = $1 - 0.7^n > 0.99$
$0.7^n < 0.01$
$n > \frac{\log 0.01}{\log 0.7} = 12.91$
Answer: n = 13