Day 175 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cot\theta = \frac{1}{2}$ and θ is acute,

find the exact value of $\operatorname{cosec}\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan\theta = 2$, so $\sec^2\theta = 1 + 4 = 5$

$\cot\theta = \frac{1}{2}$, so $\operatorname{cosec}^2\theta = \frac{5}{4}$ and $\operatorname{cosec}\theta = \frac{\sqrt{5}}{2}$

Answer: $\frac{\sqrt{5}}{2}$

2 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from ground level with speed 39.2 m s^{-1}.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top v = 0: $0 = 39.2^2 - 2 \times 9.8 \times H$

$H = 1536.64 \div 19.6 = 78.4$

(b) s = 0: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$

Answer: (a) 78.4 m (b) 8 s

3 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:

(a) y = 3f(x)

(b) y = f(2x)

(a) y = 3f(x) is a stretch, scale factor 3, parallel to the y-axis.

(b) y = f(2x) is a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

Answer: (a) stretch, scale factor 3, parallel to the y-axis (b) stretch, scale factor $\frac{1}{2}$, parallel to the x-axis

4 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (2x^2 + \frac{4}{x})\,dx$.

$\int (2x^2 + \frac{4}{x})\,dx = \frac{2x^3}{3} + 4\ln x$

$= [\frac{2x^3}{3} + 4\ln x]_1^{\,2}$

$= \ln 16 + \frac{14}{3}$

Answer: $\ln 16 + \frac{14}{3}$

5 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t$, $y = 2t^2 - 4t - 2$, $t \in \mathbb{R}$.

Find a Cartesian equation of the curve in the form y = f(x).

From the first equation: $t = \frac{x + 0}{2}$

Substitute t into $y = 2t^2 - 4t - 2$ and expand.

$y = \frac{1}{2}x^2 - 2x - 2$

Answer: $y = \frac{1}{2}x^2 - 2x - 2$

Day 175 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (-3)(3) + (4)(0) + (2)(-4) = -17$

$|\mathbf{a}| = \sqrt{29}$, $|\mathbf{b}| = \sqrt{25}$

$\cos\theta = \frac{-17}{\sqrt{29}\sqrt{25}} = -0.6314$

$\theta = 129.2°$

Answer: 129.2°

7 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.2. In a sample of 30 students, 10 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.2.

H_0: $p = 0.2$, H_1: $p \neq 0.2$. Under H_0, $X \sim B(30, 0.2)$

$P(X \geq 10) = 1 - P(X \leq 9) = 0.0611$

Two-tailed: compare with 0.025. 0.0611 > 0.025, so do not reject H_0

There is insufficient evidence that the probability is different from 0.2

Answer: do not reject H_0: insufficient evidence that the probability is different from 0.2

8 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(2x + \frac{1}{x})^8$.

The general term is $C(8, k)(2x)^{8-k}(1/x)^k$, with power of x equal to $8 - 2k$

Independent of x: $8 - 2k = 0$, so $k = 4$

Term = $70 \times 2^4 \times 1^4 = 1120$

Answer: 1120

9 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the test scores for two groups.

(a) Find the interquartile range of each group.

(b) Compare the two distributions.

(a) Class A: $35 - 25 = 10$. Class B: $50 - 35 = 15$

(b) Medians: 30 (Class A) and 45 (Class B), so Class B has the higher median

IQRs: Class B has the larger IQR, so its scores are more spread out

Answer: (a) 10 and 15 (b) Class B has the higher median; Class B is more spread out (larger IQR), Class A more consistent

10 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 26 m s^{-1} and vertical velocity component 34.3 m s^{-1} upwards.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 34.3\,t - 4.9t^2$, so $t = 34.3 \div 4.9$

(b) Range = horizontal speed × time = 26 × 7

Answer: (a) 7 s (b) 182 m