Day 174 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-8 \leq 4x - 5 \leq -3$.

Subtract -5 from each part: $-3 \leq 4x \leq 2$

Divide each part by 4: $-\frac{3}{4} \leq x \leq \frac{1}{2}$

Answer: $-\frac{3}{4} \leq x \leq \frac{1}{2}$

2 Standardising to compare (All boards)

Marks are normal: Biology N(48, 6^2), Chemistry N(50, 5^2).
Anna scores 55.5 in Biology and 47.5 in Chemistry. Which is her better performance, relatively?

z-score in Biology = $\frac{55.5 - 48}{6} = 1.25$

z-score in Chemistry = $\frac{47.5 - 50}{5} = -0.500$

The higher z-score is in Biology

Answer: Biology

3 Trig equations in radians (All boards)

Solve $\sin 2x = -0.5$ for $0 \leq x \leq \pi$,
giving answers to 3 significant figures.

Let $\theta = 2x$, so $\sin \theta = -0.5$ for $0 \leq \theta \leq 2\pi$
Calculator: $\theta = -0.5236$, then use the symmetry of the graph
$\theta = 3.665, 5.760$
$x = \frac{\theta}{2}$
$x = 1.833, 2.880$
Answer: x = 1.83, 2.88

4 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2 - 2t)\mathbf{i} + (t^3 + 3t)\mathbf{j}$ metres.
Find the velocity when t = 3, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t - 2)\mathbf{i} + (3t^2 + 3)\mathbf{j}$
$t = 3$: $\mathbf{v} = (16)\mathbf{i} + (30)\mathbf{j}$
Speed = $\sqrt{(16^2 + 30^2)}$
Answer: $\mathbf{v} = (16)\mathbf{i} + (30)\mathbf{j}$ m s^{-1}; speed 34 m s^{-1}

5 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = \ln x$ to find the exact value of

$\int_1^e \frac{(\ln x)^3}{x}\,dx$.

$\frac{du}{dx} = \frac{1}{x}$, so $\frac{1}{x}\,dx = du$
Limits: $x = 1$ gives $u = 0$, $x = e$ gives $u = 1$
$\int_0^1 u^3\,du = \left[\frac{u^4}{4}\right]_0^1$
$= \frac{1}{4}$

Answer: $\frac{1}{4}$

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6 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

$P(A) = 0.48 + 0.12 = 0.6$ and $P(A \cap B) = 0.12$
Independent: $P(A \cap B) = P(A)P(B)$, so $P(B) = 0.12 \div 0.6 = 0.2$
$x = P(B) - P(A \cap B) = 0.2 - 0.12 = 0.08$
$y = 1 - (0.48 + 0.12 + 0.08) = 0.32$
Answer: $x = 0.08$, $y = 0.32$

7 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve $y = f(x)$, where f is a cubic. The curve meets the axes at the labelled points. Find $f(x)$ in factorised form.

x-intercepts at $x = -4$, 1 and 3, so $y = k(x + 4)(x - 1)(x - 3)$
The curve passes through (0, 24): $k \times 4 \times (-1) \times (-3) = 24$
So $k = 2$
Answer: $f(x) = 2(x + 4)(x - 1)(x - 3)$

8 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (t^2 + 5)\mathbf{i} + (2t - 4)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$2t - 4 = 0$, so $t = 2$
$\mathbf{v} = (1 \times 2^2 + 5)\mathbf{i} = 9\mathbf{i}$
Answer: $t = 2$ s, speed 9 m s^{-1}

9 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(2r - 2) = 56$, find the value of n.

Sum $= \frac{n}{2}(0 + 2n - 2) = 56$

$2n^2 - 2n - 112 = 0$
$2(n - 8)(n + 7) = 0$
n must be positive, so $n = 8$
Answer: $n = 8$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ -6 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-2 + 1\lambda = 0 + 0\mu$
j: $-1 + (-1)\lambda = -6 + 3\mu$
Solving i and j: $\lambda = 2$, $\mu = 1$
k check: $-2 + 0(2) = -2$ and $-4 + 2(1) = -2$, equal

Point: $\begin{pmatrix} 0 \\ -3 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{0} \\ \mathbf{-3} \\ \mathbf{-2} \end{pmatrix}$