Day 173 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-10 \leq 5x - 4 < 3$.

Subtract -4 from each part: $-6 \leq 5x < 7$

Divide each part by 5: $-\frac{6}{5} \leq x < \frac{7}{5}$

Answer: $-\frac{6}{5} \leq x < \frac{7}{5}$

2 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 2\tan 4x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 8\sec^2 4x$

At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = 32$

Answer: $\frac{dy}{dx} = 8\sec^2 4x$; gradient 32

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (3, 8) and (4, −5).

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{3+4}{2}, \frac{8+(-5)}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right)$

(b) Gradient $= \frac{-13}{1} = -13$

$y - 8 = -13(x - 3)$

Multiply out and collect terms: $13x + y - 47 = 0$

Answer: (a) $\left(\frac{7}{2}, \frac{3}{2}\right)$ (b) $13x + y - 47 = 0$

4 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 10 kg hangs horizontally from two vertical strings attached 1 m from A and 1 m from B.

A particle of mass 2 kg is attached 1.5 m from A. Take $g = 9.8$ m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 10\,g(1) + 2\,g(0.5)$

$T_2 = 53.9$

$T_1 + T_2 = 12\,g$ so $T_1 = 117.6 - 53.9$

Answer: Near A: 63.7 N; near B: 53.9 N

5 Probability without replacement (All boards)

A bag has 7 green and 9 yellow beads; two are taken without replacement.

(a) Find P(same colour).

(b) Given they are the same colour, find P(both green).

(a) P(green green) $= \frac{7}{16} \times \frac{6}{15} = \frac{7}{40}$

P(yellow yellow) $= \frac{9}{16} \times \frac{8}{15} = \frac{3}{10}$

P(same) $= \frac{7}{40} + \frac{3}{10} = \frac{19}{40}$

(b) $P = \frac{7}{40} \div \frac{19}{40} = \frac{7}{19}$

Answer: (a) $\frac{19}{40}$ (b) $\frac{7}{19}$

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6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = (2 + y)e^{-x}$,

given that $y = 2$ when $x = 0$.

$\frac{1}{2+y}\,dy = e^{-x}\,dx$, so $\ln(2 + y) = -e^{-x} + c$

$x = 0$: $\ln 4 = -1 + c$, so $c = 1 + \ln 4$

$2 + y = 4\,e^{1 - e^{-x}}$, so $y = 4\,e^{1 - e^{-x}} - 2$

Answer: $y = 4e^{1 - e^{-x}} - 2$

7 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 4)(x + 3)(x - 3) \geq 0$.

Critical values: $x = -4$, $x = -3$, $x = 3$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $-4 \leq x \leq -3$ or $x \geq 3$

Answer: $-4 \leq x \leq -3$ or $x \geq 3$

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from level ground with horizontal velocity component 19 m s^{-1} and vertical velocity component 39.2 m s^{-1} upwards.

Take $g = 9.8$ m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$

(b) Range = horizontal speed × time = 19×8

Answer: (a) 8 s (b) 152 m

9 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.15, find the least number of components needed for the probability of at least one success to exceed 0.95.

P(at least one) = $1 - 0.85^n > 0.95$

$0.85^n < 0.05$

$n > \frac{\log 0.05}{\log 0.85} = 18.43$

Answer: $n = 19$

10 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 12 cm^{2}s^{-1}.

Find the rate of increase of the side length when the side is 2 cm.

$A = x^2$, so $\frac{dA}{dt} = 2x\frac{dx}{dt}$

$12 = 2 \times 2 \times \frac{dx}{dt}$

$\frac{dx}{dt} = 3$

Answer: 3 cm/s