Day 172 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exponential growth and decay models (All boards)

The mass of a radioactive sample, M grams, after t days is $M = 80\,e^{-0.25t}$.
Find (a) the half-life, (b) the rate of decrease in mass when t = 5, each to 3 s.f.

(a) $80\,e^{-0.25t} = 40$: $e^{-0.25t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.25} = 2.773$ days

(b) $\frac{dM}{dt} = -0.25 \times 80\,e^{-0.25t} = -20\,e^{-0.25t}$

At t = 5: $\frac{dM}{dt} = -20\,e^{-1.25} = -5.730$

so mass is decreasing at 5.73 g per day

Answer: (a) 2.77 days (b) 5.73 g/day

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5x + 10 \geq -2x - 3$.

$7x \geq -13$

Divide by 7: $x \geq -\frac{13}{7}$

Answer: $x \geq -\frac{13}{7}$

3 Work-energy: sliding to rest (CIE)

A block of mass 2 kg is projected along a rough horizontal surface with speed 8.4 m s^{-1} and comes to rest after sliding 12 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 2 \times 9.8 \times 12 = \frac{1}{2} \times 2 \times 8.4^2$

$235.2\,\mu = 70.56$

Answer: 0.3

4 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cos\theta + 3\sin\theta \equiv \sqrt{10}\cos(\theta - 71.6°)$,
find the greatest value of $\frac{1}{(6 + \cos\theta + 3\sin\theta)}$.

Denominator is smallest when $\sqrt{10}\cos(\theta - 71.6°) = -\sqrt{10}$

Smallest denominator = $6 - \sqrt{10}$

Greatest value = $\frac{1}{(6 - \sqrt{10})}$, which rationalises to $\frac{\sqrt{10} + 6}{26}$

Answer: $\frac{\sqrt{10} + 6}{26}$

5 Binomial one-tailed test (All boards)

A seed supplier says that 0.25 of seeds germinate. In a sample of 30 seeds, 4 germinate.
Test, at the 1% level, whether the true proportion is lower than claimed.

H_0: p = 0.25, H_1: p < 0.25. Let X ~ B(30, 0.25) under H_0

$P(X \leq 4) = 0.0979$

0.0979 > 0.01, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

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6 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $21\cos\theta + 20\sin\theta \equiv 29\cos(\theta - 43.6°)$,
solve $21\cos\theta + 20\sin\theta = 2.5$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$29\cos(\theta - 43.6°) = 2.5$, so $\cos(\theta - 43.6°) = \frac{2.5}{29}$

$\theta - 43.6° = \pm 85.1°$

$\theta = 43.6° + 85.1° = 128.7°$

$\theta = 43.6° - 85.1° = -41.5°$, so $\theta = 318.5°$

Answer: $\theta = 128.7°, 318.5°$

7 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{x^2 - 13x + 34}{(x-2)(x-5)}$ in the form $P + \frac{Q}{(x-2)} + \frac{R}{(x-5)}$.

$x^2 - 13x + 34 = 1(x-2)(x-5) + (-6x + 24)$, so $P = 1$

$\frac{x^2 - 13x + 34}{(x-2)(x-5)} \equiv 1 + \frac{Q}{(x-2)} + \frac{R}{(x-5)}$

$x = 2$ gives $Q = -4$; $x = 5$ gives $R = -2$

Answer: $1 + -\frac{4}{(x-2)} - \frac{2}{(x-5)}$

8 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.25. In a sample of 40 students, 14 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$. Under H_0, $X \sim B(40, 0.25)$

$P(X \geq 14) = 1 - P(X \leq 13) = 0.1032$

Two-tailed: compare with 0.025. $0.1032 > 0.025$, so do not reject H_0

There is insufficient evidence that the probability is different from 0.25

Answer: do not reject H_0: insufficient evidence that the probability is different from 0.25

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x - 8$ meets the circle $x^2 + (y+1)^2 = 169$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x - 8$: $x^2 + (-x - 7)^2 = 169$

$x^2 + 7x - 60 = 0$

$(x - 5)(x + 12) = 0$, so $x = -12$ or $x = 5$

$y = -x - 8$: A = $(-12, 4)$, B = $(5, -13)$

$AB^2 = 17^2 + (-17)^2 = 578$

Answer: A$(-12, 4)$, B$(5, -13)$, AB $= 17\sqrt{2}$

10 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected horizontally at 147 m s^{-1} from a point high above the ground.

Take $g = 9.8$ m s^{-2}.

Find the speed and the direction of motion 20 s later, to 1 d.p. below the horizontal.

Vertical speed $= 9.8 \times 20 = 196$

Speed $= \sqrt{(147^2 + 196^2)} = 245$

$\tan\alpha = 196 \div 147$, $\alpha = 53.1°$

Answer: 245 m s^{-1} at 53.1° below horizontal