Day 171 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 117; systematic sample of size 13.
(a) Find the sampling interval.
(b) The first item is number 5. Find the 6-th item.

(a) Interval = 117 ÷ 13 = 9
(b) 6-th item = 5 + 5 × 9 = 50
Answer: interval 9; item number 50

2 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(3\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-7\mathbf{i} + 6\mathbf{j})$ m.
Find (a) its position vector when t = 4, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-7\mathbf{i} + 6\mathbf{j}) + 4(3\mathbf{i} - \mathbf{j}) = (5\mathbf{i} + 2\mathbf{j})$
(b) $\mathbf{j}$ component zero: 6 − t = 0, so t = 6
Answer: (a) $(5\mathbf{i} + 2\mathbf{j})$ m (b) t = 6 s

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "The sum of two irrational numbers is always irrational."
Use the numbers $2 + \sqrt{2}$ and $6 - \sqrt{2}$ to show this is false.

$(2 + \sqrt{2}) + (6 - \sqrt{2}) = 8$
Both numbers are irrational, but 8 is rational, so the statement is false.
Answer: sum = 8, which is rational

4 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x - \operatorname{cosec} x - 5 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $\operatorname{cosec}^2 x - \operatorname{cosec} x - 6 = 0$
$(\operatorname{cosec} x - 3)(\operatorname{cosec} x + 2) = 0$, so $\operatorname{cosec} x = 3$ or -2
$\operatorname{cosec} x = 3$, so $\sin x = \frac{1}{3}$: x = 19.5°, 160.5°
$\operatorname{cosec} x = -2$, so $\sin x = -\frac{1}{2}$: x = 210.0°, 330.0°
Answer: x = 19.5°, 160.5°, 210.0°, 330.0°

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$3x^3 - 4x^2 - 12x + 14$ is divided by $(x - 2)$.

$3x^2 \times (x - 2) = 3x^3 - 6x^2$, leaving $2x^2 - 12x + 14$
$2x \times (x - 2) = 2x^2 - 4x$, leaving $-8x + 14$
$-8 \times (x - 2) = -8x + 16$, leaving -2
Quotient $3x^2 + 2x - 8$, remainder -2
Answer: quotient $3x^2 + 2x - 8$, remainder -2

6 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 40, is a bar 4 cm wide, 5 cm tall.
(a) Find the bar height for width 20, frequency 45.
(b) Find the frequency of a 1 cm by 3 cm bar.

Area of bar ∝ frequency: 4 × 5 = 20 cm^2 represents 40
so 1 unit of frequency = 0.5 cm^2
(a) Width = 4 cm, area = 0.5 × 45 = 22.5 cm^2, height = 22.5 ÷ 4 = 5.625 cm
(b) Area = 1 × 3 = 3 cm^2, frequency = 3 ÷ 0.5 = 6
Answer: (a) 5.625 cm (b) 6

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7 Area under a curve (All boards)

The diagram shows the curve $y = x(5 - x)$ and the line $y = x$.
Find the exact area of the shaded region R.

Intersect: $x(5 - x) = x$, so $x = 0$ or $x = 4$

Area $= \int_0^4 (x(5 - x) - x)\,dx = \int_0^4 (4x - x^2)\,dx$

$= [2x^2 - \frac{x^3}{3}]_0^4 = \frac{32}{3}$

Answer: $\frac{32}{3}$

8 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 20t - 2t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 20 - 4t$
Maximum speed when $a = 0$: $t = 20 \div 4 = 5$
$v = 20 \times 5 - 2 \times 5^2$
Answer: 50 m s^{-1} at t = 5 s

9 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 2$ and θ is acute,
find the exact value of $\operatorname{cosec}\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = 2$, so $\sec^2\theta = 1 + 4 = 5$
$\cot\theta = \frac{1}{2}$, so $\operatorname{cosec}^2\theta = \frac{5}{4}$ and $\operatorname{cosec}\theta = \frac{\sqrt{5}}{2}$

Answer: $\frac{\sqrt{5}}{2}$

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 2x + 4$.

C is translated by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

Find its new equation.

Translation: replace x by $(x - h)$ and add k to y.

When translated by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$: $y = x^2 + 5$

Answer: $y = x^2 + 5$