**Day 170** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 12 and the common difference is 5.
The sum of the first n terms is 345. Find the value of n.

$S_n = \frac{n}{2}(2 \times 12 + (n - 1) \times 5) = 345$

$5n^2 + 19n - 690 = 0$
$(n - 10)(5n + 69) = 0$
n must be positive, so n = 10
**Answer: n = 10**

## 2 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg rests on a rough horizontal floor, with coefficient of friction 0.4. A horizontal force of 16 N is applied to the box.
Take $g = 9.8$ m s$^{-2}$.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 5 \times 9.8 = 49$, maximum friction $\mu R = 0.4 \times 49 = 19.6$
$16 < 19.6$ so the box stays at rest
Friction balances the applied force: F = 16
**Answer: Does not move; friction = 16 N**

## 3 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.25 × 0.7 = 0.175
P(not E and F) = 0.75 × 0.4 = 0.3
Exactly one: 0.175 + 0.3 = 0.475
**Answer: 0.475**

## 4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=7}^{17}(5r + 3)$.

There are $17 - 7 + 1 = 11$ terms
First term = 38, last term = 88
$S = \frac{11}{2}(38 + 88) = 693$
**Answer: 693**

## 5 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $2\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.
(b) Hence find its maximum value and the smallest $\theta \geq 0$ at which it occurs. Give α to 1 d.p.

$R = \sqrt{2^2 + 3^2} = \sqrt{13}$, $\tan\alpha = \frac{3}{2}$ so $\alpha = 56.3°$
Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$
**Answer: (a) $\sqrt{13}\cos(\theta - 56.3°)$ (b) max $\sqrt{13}$ at $\theta = 56.3°$**

## 6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of $x^2$ in the expansion of
$(2 + 3x)(2 + x)^5$.

$(2 + x)^5$ has $x^1$ coefficient 80 and $x^2$ coefficient 80
$x^2$ term of the product = 2 × 80 + 3 × 80
= 400
**Answer: 400**

**Day 170** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 7 Binomial: smallest n (All boards)

Given that each attempt at a task succeeds independently with probability 0.4, find the least number of attempts needed for the probability of at least one success to exceed 0.95.

P(at least one) = $1 - 0.6^n > 0.95$

$0.6^n < 0.05$

$n > \frac{\log 0.05}{\log 0.6} = 5.864$

**Answer: n = 6**

## 8 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 2 kg hangs in equilibrium on a light string inclined at $\alpha$ to the vertical, held by a horizontal force P N, as shown. $\tan \alpha = \frac{12}{5}$.

Take g = 9.8 m $s^{-2}$.

Find the tension in the string and the value of P.

$\cos \alpha = 5/13$, $\sin \alpha = 12/13$

Vertical: $T \cos \alpha = mg = 19.6$, so $T = 19.6 \times 13/5$

Horizontal: $P = T \sin \alpha = 50.96 \times 12/13$

**Answer: T = 50.96 N; P = 47.04 N**

## 9 Double angle equations (All boards)

Solve $\cos 2x + 3 \cos x = 1$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2\cos^2 x - 1 + 3 \cos x - 1 = 0$

$(2 \cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$: x = 60.0°, 300.0°

$\cos x = -2$ has no solutions

**Answer: x = 60.0°, 300.0°**

## 10 Locating roots: change of sign (All boards)

The function $f(x) = e^x - 7x$.

Show that f(x) = 0 has a root between x = 3 and x = 3.5.

f(3) = −0.914

f(3.5) = 8.62

f is continuous and changes sign, so there is a root in the interval

**Answer: f(3) < 0 and f(3.5) > 0: sign change**