Day 17 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(3 + 4x)^7$ in ascending powers of x, up to and including the term in x^3.

$(3 + 4x)^7 = 3^7 + 7 \times 3^6(4x) + 21 \times 3^5(4x)^2 + 35 \times 3^4(4x)^3$

$= 2187 + 20412x + 81648x^2 + 181440x^3 + ...$

Answer: $2187 + 20412x + 81648x^2 + 181440x^3 + ...$

2 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0e^{kt}$, where t is in hours. Initially N = 500 and after 5 hours N = 750.
Find (a) k to 3 s.f., (b) when N reaches 2500, to 3 s.f.

$N_0 = 500$, so $750 = 500\ e^{5k}$, $e^{5k} = 1.5$

(a) $k = \frac{1}{5} \ln 1.5 = 0.08109$

(b) $500\ e^{kt} = 2500$, so $t = \frac{\ln 5}{k} = 19.85$ hours

Answer: (a) k = 0.0811 (b) t = 19.8 hours

3 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 80 kg stands on the floor of a lift. The lift accelerates downwards at 0.25 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 80 \times 9.8 = -80 \times 0.25$

$R = 80(9.8 - 0.25)$

Answer: 764 N

4 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{35}{\sqrt{7}} + \sqrt{112}$, giving your answer in the form $k\sqrt{7}$.

$\frac{35}{\sqrt{7}} = \frac{35\sqrt{7}}{7} = 5\sqrt{7}$

$\sqrt{112} = 4\sqrt{7}$

$5\sqrt{7} + 4\sqrt{7} = 9\sqrt{7}$

Answer: $9\sqrt{7}$

5 Inequalities with exponents and logs (All boards)

£1000 is invested at 8% compound interest per year.
Find the least whole number of years for the investment to exceed £3000.

$1000 \times 1.08^n > 3000$, so $1.08^n > 3$

$n > \frac{\ln 3}{\ln 1.08} = 14.27$

The least whole number is n = 15

Answer: 15 years

6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 + 2x)(1 - 2x)^6$.

$(1 - 2x)^6$ has x^1 coefficient -12 and x^2 coefficient 60

x^2 term of the product $= 1 \times 60 + 2 \times (-12)$

$= 36$

Answer: 36

7 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{216}$ for $0 \le x \le 6$, and 0 otherwise.

(a) Find $P(X > 5)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{216}$

(a) $P(X > 5) = 1 - F(5) = \frac{91}{216}$

(b) $F(m) = 0.5$: $\frac{m^3}{216} = \frac{1}{2}$

$m = 4.76$

Answer: (a) $\frac{91}{216}$ (b) 4.76

8 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 9 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{3}{5}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 9$, so $P \times \frac{3}{5} = 9$

$P = 15$

Resolve vertically: $Q = P\sin\theta = 15 \times \frac{4}{5}$

Answer: P = 15 N; Q = 12 N

9 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{4}} (3\cos^2 2x - 1)\,dx$.

Use $\cos^2 2x = \frac{1}{2}(1 + \cos 4x)$

$\int (3\cos^2 2x - 1)\,dx = \frac{x}{2} + \frac{3\sin 4x}{8}$

$= \left[\frac{x}{2} + \frac{3\sin 4x}{8}\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8}$

Answer: $\frac{\pi}{8}$

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 6 g. Claimed mean 60 g; 9 packets have mean 62 g.
Test at the 5% level whether the true mean differs from 60.

$H_0: \mu = 60$, $H_1: \mu \neq 60$. Under H_0, $\overline{X} \sim N(60, \frac{6^2}{9})$

$z = \frac{62 - 60}{6/\sqrt{9}} = 1.000$

$P(Z > 1.000) = 0.1587 > 0.025$

Do not reject H_0: there is insufficient evidence that the mean has changed

Answer: do not reject H_0: insufficient evidence of a change in the mean