Day 169 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van of mass 1200 kg moves along a straight horizontal road. The engine produces a driving force of 3000 N and the resistance to motion is 600 N.

Find the acceleration of the van.

Resultant force = 3000 − 600 = 2400 N

F = ma: 2400 = 1200 a

Answer: 2 m s^{-2}

2 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the masses, m grams, of 7 eggs gives $\Sigma m = 229$ and $\Sigma m^2 = 7827$.

Find the mean and the standard deviation (3 s.f.).

Mean = 229 ÷ 7 = 32.7

Variance = $\frac{7827}{7} - (32.714)^2 = 47.92$

Standard deviation = $\sqrt{47.92} = 6.92$

Answer: mean 32.7, standard deviation 6.92

3 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 25}$.

$\frac{dy}{dx} = \frac{25 - x^2}{(x^2 + 25)^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $-(x - 5)(x + 5) = 0$

$x = -5$ or $x = 5$

Answer: x = −5, x = 5

4 Work-energy: sliding to rest (CIE)

A block of mass 5 kg is projected along a rough horizontal surface with speed 14 m s^{-1} and comes to rest after sliding 25 m.

Take g = 9.8 m s^{-2}.

Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 5 \times 9.8 \times 25 = \frac{1}{2} \times 5 \times 14^2$

$1225\,\mu = 490$

Answer: 0.4

5 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the trapezium rule with 4 strips.

Use it to estimate $\int_0^2 e^{-x^2}\,dx$, to 3 s.f.

$h = \frac{2 - 0}{4} = 0.5$

x: 0, 0.5, 1, 1.5, 2

y: 1.000, 0.7788, 0.3679, 0.1054, 0.01832

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [1.000 + 0.01832 + 2 \times 1.252] = 0.8806$

Answer: 0.881

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6 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A charity raises £150 in week 1. In each following week it raises £15 more than in the previous week.
(a) Find the amount raised in week 8.
(b) Find the total for the first 13 weeks.

(a) Arithmetic sequence with a = 150, d = 15
$u_8 = 150 + 7 \times 15 = £255$

(b) $S_{13} = \frac{13}{2}(2 \times 150 + 12 \times 15) = £3{,}120$

Answer: (a) £255 (b) £3,120

7 Two independent binomials (All boards)

X ~ B(12, 0.3) and Y ~ B(12, 0.6) are independent.
Find P(X = 3 and Y ≤ 8), to 3 significant figures.

P(X = 3) = 0.2397
P(Y ≤ 8) = 0.7747
Independent, so multiply: 0.2397 × 0.7747 = 0.186
Answer: 0.186

8 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 9 cm. Angle AOB = 0.8 radians.
Find the area of the segment, to 3 s.f.

Sector area = $\frac{1}{2} \times 9^2 \times 0.8 = 32.4$

Triangle area = $\frac{1}{2} \times 9^2 \times \sin 0.8 = 29.05$

Segment = sector − triangle = 3.347 cm^2
Answer: 3.35 cm^2

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-3\sqrt{2}}{2+\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by the conjugate $(2 - \sqrt{2})$:
Denominator: $(2+\sqrt{2})(2-\sqrt{2}) = 2^2 - 1^2 \times 2 = 2$
Numerator: $(-3\sqrt{2})(2-\sqrt{2}) = 6 - 6\sqrt{2}$
So the fraction = $\frac{6-6\sqrt{2}}{2} = 3 - 3\sqrt{2}$
Answer: $3 - 3\sqrt{2}$

10 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
2x + y = 15
$x^2 + y^2 = 125$

$y = -2x + 15$; substitute: $x^2 + (-2x + 15)^2 = 125$
$5x^2 - 60x + 100 = 0$, so $x^2 - 12x + 20 = 0$
$(x - 2)(x - 10) = 0$, so x = 2 or x = 10
y = 15 − 2x gives y = 11 and y = −5
Answer: x = 2, y = 11 and x = 10, y = −5