Day 168 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^e (3x^2 + \frac{3}{x})\,dx$.

$\int (3x^2 + \frac{3}{x})\,dx = x^3 + 3\ln x$

$= [x^3 + 3\ln x]_1^e$

$= 2 + e^3$

Answer: $2 + e^3$

2 Union, conditional probability and independence (All boards)

P(A) = 0.7, P(B) = 0.2, P(A ∩ B) = 0.14.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.7 + 0.2 - 0.14 = 0.76$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.14}{0.2} = 0.7$

(c) $P(A)P(B) = 0.7 \times 0.2 = 0.14$, which equals $P(A \cap B)$

Answer: (a) 0.76 (b) 0.7 (c) independent

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 - 11x > -5$.

Rearrange: $2x^2 - 11x + 5 > 0$
Factorise: $(x - 5)(2x - 1) > 0$

Critical values: $x = \frac{1}{2}$ and $x = 5$

Sketch the ∪-shaped parabola: $x < \frac{1}{2}$ or $x > 5$

Answer: $x < \frac{1}{2}$ or $x > 5$

4 Binomial critical region (All boards)

$X \sim B(50, p)$. H_0: $p = 0.4$ is to be tested against H_1: $p < 0.4$ at the 1% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(50, 0.4)$
$P(X \leq 11) = 0.0057 \leq 0.01$
$P(X \leq 12) = 0.0133 > 0.01$
The critical region is $X \leq 11$
Answer: $X \leq 11$; actual significance level 0.57%

5 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a maximum point at (−2, −3).
Find the turning point on the curve:

(a) $y = f(\frac{x}{4})$ (b) $y = 2f(x)$

(c) $y = f(-x)$

(a) $y = f(\frac{x}{4})$: the point (−2, −3) maps to (−8, −3)

(b) $y = 2f(x)$: the point (−2, −3) maps to (−2, −6)
(c) $y = f(-x)$: the point (−2, −3) maps to (2, −3)
Answer: (a) (−8, −3) (b) (−2, −6) (c) (2, −3)

6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = 2y^2e^x$

and y = 0.5 when x = 0.

$\frac{1}{y^2}\,dy = 2e^x\,dx$, so $-\frac{1}{y} = 2e^x + c$

x = 0, y = $\frac{1}{2}$: $-2 = 2 + c$, so $c = -4$

$\frac{1}{y} = 4 - 2e^x$, so $y = \frac{1}{4 - 2e^x}$

Answer: $\mathbf{y = \frac{1}{(4 - 2e^x)}}$

7 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(16 - x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 16: $(16 - x)^{-\frac{1}{2}} = 16^{-\frac{1}{2}}(1 + -\frac{1}{16}x)^{-\frac{1}{2}} = \frac{1}{4}(1 + -\frac{1}{16}x)^{-\frac{1}{2}}$

Expand $(1 + -\frac{1}{16}x)^{-\frac{1}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by $\frac{1}{4}$: coefficients of 1, x, x^2, x^3 are $\frac{1}{4}$, $\frac{1}{128}$, $\frac{3}{8192}$, $\frac{5}{262144}$

Valid when $|\frac{1}{16}x| < 1$, that is $|x| < 16$

Answer: (a) $\mathbf{\frac{1}{4} + \frac{1}{128}x + \frac{3}{8192}x^2 + \frac{5}{262144}x^3 + ...}$ **(b)** $\mathbf{|x| < 16}$

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 10°C: $\frac{dT}{dt} = -k(T - 10)$. Initially T = 70 and after 6 minutes T = 55.

Find k to 3 s.f. and the time to cool to 20°C, to 3 s.f.

$\frac{1}{T - 10}\,dT = -k\,dt$, so $\ln(T - 10) = -kt + c$, i.e. $T = 10 + 60e^{-kt}$

t = 6: $55 = 10 + 60e^{-6k}$, so $e^{-6k} = \frac{3}{4}$

$k = \frac{1}{6}\ln\frac{60}{45} = 0.04795$

$20 = 10 + 60e^{-kt}$: $t = \frac{1}{k}\ln\frac{60}{10} = 37.37$

Answer: k = 0.0479, t = 37.4 minutes

9 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its acceleration is $3t + 4$ m s^{-2}.
When t = 0 the velocity is 2 m s^{-1}. Find the velocity when t = 4.

$v = \int (3t + 4)\,dt = \frac{3}{2}t^2 + 4t + c$

t = 0, v = 2 so c = 2
t = 4: v = 42

Answer: 42 m s^{-1}

10 Work-energy: sliding to rest (CIE)

A block of mass 5 kg is projected along a rough horizontal surface with speed 8.4 m s^{-1} and comes to rest after sliding 12 m.

Take g = 9.8 m s^{-2}.

Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 5 \times 9.8 \times 12 = \frac{1}{2} \times 5 \times 8.4^2$

$588\mu = 176.4$

Answer: 0.3