Day 167 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + px^2 + qx - 6$.
When f(x) is divided by (x + 2) the remainder is 10, and when divided by (x − 2) the remainder is 2. Find p and q.

$f(-2) = 10$: $-8 + 4p - 2q - 6 = 10$, so $4p - 2q = 24$
$f(2) = 2$: $8 + 4p + 2q - 6 = 2$, so $4p + 2q = 0$
Solving simultaneously: $p = 3$, $q = -6$
Answer: $p = 3$, $q = -6$

2 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = k(9 - x)$ for $x = 1, 2, ..., 6$.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(8 + 7 + 6 + 5 + 4 + 3) = 1$, so $33k = 1$ and $k = \frac{1}{33}$

(b) $E(X) = 1 \times \frac{8}{33} + 2 \times \frac{7}{33} + 3 \times \frac{2}{11} + 4 \times \frac{5}{33} + 5 \times \frac{4}{33} + 6 \times \frac{1}{11} = \frac{98}{33}$

$E(X^2) = \frac{126}{11}$, so $Var(X) = \frac{126}{11} - (\frac{98}{33})^2 = \frac{2870}{1089}$

Answer: (a) $k = \frac{1}{33}$ (b) $E(X) = \frac{98}{33}$, $Var(X) = \frac{2870}{1089}$

3 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{4x - 4}$ for $x \geq 1$.
Find $f^{-1}(x)$ and state its domain.

Let $y = \sqrt{4x - 4}$. Then $y^2 = 4x - 4$
$x = \frac{y^2 + 4}{4}$
$f(x) \geq 0$, so the domain of f^{-1} is $x \geq 0$.

Answer: $f^{-1}(x) = \frac{x^2 + 4}{4}$, domain $x \geq 0$

4 Fixed-point iteration (All boards)

(a) Show that $x^2 - 4x - 7 = 0$ can be written as $x = \sqrt{4x + 7}$.
(b) Use $x_{n+1} = \sqrt{4x_n + 7}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = 4x + 7$, so $x = \sqrt{4x + 7}$
(b) $x_1 = 4.3589$
$x_2 = 4.9432$
$x_3 = 5.1743$
Answer: $x_1 = 4.3589$, $x_2 = 4.9432$, $x_3 = 5.1743$

5 Work-energy: speed after falling or sliding (CIE)

A bead of mass 5 kg is released from rest 22.5 m above the ground and falls freely.
Take $g = 9.8$ m s^{-2}.
Use energy to find its speed on reaching the ground.

Loss of PE = gain of KE: $m \times 9.8 \times 22.5 = \frac{1}{2} m v^2$
$v^2 = 19.6 \times 22.5 = 441$
$v = 21$
Answer: 21 m s^{-1}

6 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 48, is a bar 2.5 cm wide, 9.6 cm tall.
(a) Find the bar height for width 20, frequency 15.
(b) Find the frequency of a 2 cm by 4 cm bar.

Area of bar $\propto$ frequency: $2.5 \times 9.6 = 24$ cm^2 represents 48
so 1 unit of frequency = 0.5 cm^2
(a) Width = 5 cm, area = $0.5 \times 15 = 7.5$ cm^2, height = $7.5 \div 5 = 1.5$ cm
(b) Area = $2 \times 4 = 8$ cm^2, frequency = $8 \div 0.5 = 16$
Answer: (a) 1.5 cm (b) 16

7 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 8 that lie between 112 and 295.

First multiple = $8 \times 15 = 120$, last multiple = $8 \times 36 = 288$
This is an arithmetic series with d = 8 and 22 terms
$S = \frac{22}{2}(120 + 288) = 4488$
Answer: 4488

8 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 4 s.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$
$R = mg\cos\alpha$, $F = \mu R$
$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(5/13 - 0.2 \times 12/13)$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 4^2$
Answer: 1.96 m s^{-2}; 15.68 m

9 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $3\sin^2x + \cos x - 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2x = 1 - \cos^2x$: $\cos x - 3\cos^2x = 0$, i.e. $3\cos^2x - \cos x = 0$
$(3\cos x - 1)(\cos x) = 0$, so $\cos x = \frac{1}{3}$ or 0
$\cos x = \frac{1}{3}$: x = 70.5°, 289.5°
$\cos x = 0$: x = 90.0°, 270.0°
Answer: x = 70.5°, 90.0°, 270.0°, 289.5°

10 Area under a curve (All boards)

The region R is bounded by the x-axis, the lines x = 1
and x = 2, and the curve $y = 2x + \frac{2}{x}$. Find the exact area of R.

Area $= \int_1^2 (2x + \frac{2}{x})\,dx = [x^2 + 2\ln x]_1^2$
$= \ln 4 + 3$
Answer: ln 4 + 3