Day 166 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(-12\mathbf{i} + 16\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(-12^2 + 16^2)} = 20$

$\tan\theta = 16 \div 12$

Answer: 20 m s^{-1}; 53.1°

2 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 11 cm and area $\frac{242\pi}{3}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{242\pi}{3}$, so $\theta = \frac{2 \times \frac{242\pi}{3}}{11^2} = \frac{4\pi}{3}$ radians

$\theta = \frac{4\pi}{3} \times \frac{180}{\pi} = 240°$

Answer: 240°

3 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{32}{243})^{-\frac{1}{5}}$.

$\frac{32}{243} = (\frac{2}{3})^5$, so the expression = $(\frac{2}{3})^{-1}$

$= \frac{3}{2}$

Answer: $\frac{3}{2}$

4 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 3 : 2, and N lies on OB with ON : NB = 2 : 3. Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{3}{5}$(b − a)

OM = a + AM = $\frac{2a}{5} + \frac{3b}{5}$

ON = $\frac{2b}{5}$, so NM = OM − ON = $\frac{2a}{5} + \frac{b}{5}$

Answer: OM = $\frac{2a}{5} + \frac{3b}{5}$, NM = $\frac{2a}{5} + \frac{b}{5}$

5 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 5 and Var(X) = 2.

Find (a) E(3X + 2),

(b) Var(3X + 2), (c) E(X^2).

(a) E(aX + b) = aE(X) + b = 3 × 5 + 2 = 17

(b) Var(aX + b) = a^{2}Var(X) = 9 × 2 = 18

(c) Var(X) = E(X^2) − [E(X)]^2, so E(X^2) = 2 + 25 = 27

Answer: (a) 17 (b) 18 (c) 27

6 Rates of change (connected rates) (All boards)

The volume of a sphere increases at a constant rate of 32 cm^3s^{-1}.
Find the rate at which the radius is increasing when r = 3 cm, to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{32}{4\pi r^2}$

When r = 3: $\frac{dr}{dt} = \frac{8}{9\pi} = 0.2829$

Answer: 0.283 cm/s

7 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A golf ball is projected from ground level with initial velocity components 35 m s^{-1} horizontally and 29.4 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 29.4^2 - 2 \times 9.8 \times H$, $H = 864.36 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.
Answer: (a) 44.1 m (b) 35 m s^{-1}

8 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.125 + 0.125 = 0.25 and $P(A \cap B) = 0.125$
Independent: $P(A \cap B) = P(A)P(B)$, so $P(B) = 0.125 \div 0.25 = 0.5$
$x = P(B) - P(A \cap B) = 0.5 - 0.125 = 0.375$
$y = 1 - (0.125 + 0.125 + 0.375) = 0.375$
Answer: x = 0.375, y = 0.375

9 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + 9n + 4$ is even.

Let n = 2k + 1, where k is an integer.
$n^2 = 4k^2 + 4k + 1$
9n = 18k + 9
Total = $4k^2 + 22k + 14$
4k + 22 is even and 14 is even, so the total is even.
Answer: the expression is even

10 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$2x + y = -15$
$x^2 + y^2 = 50$

$y = -2x - 15$; substitute: $x^2 + (-2x - 15)^2 = 50$
$5x^2 + 60x + 175 = 0$, so $x^2 + 12x + 35 = 0$
$(x + 5)(x + 7) = 0$, so $x = -5$ or $x = -7$
$y = -15 - 2x$ gives $y = -5$ and $y = -1$
Answer: x = −7, y = −1 and x = −5, y = −5