Day 165 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of:
(a) $y = -f(x)$
(b) $y = f(x - 1)$

(a) $y = -f(x)$ is a reflection in the x-axis.

(b) $y = f(x - 1)$ is a translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Answer: (a) reflection in the x-axis (b) translation $\begin{pmatrix} \mathbf{1} \\ \mathbf{0} \end{pmatrix}$

2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 60 kg stands on the floor of a lift. The lift accelerates downwards at 2 m s^{-2}.
Take $g = 9.8$ m s^{-2}.
Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 60 \times 9.8 = -60 \times 2$
$R = 60(9.8 - 2)$
Answer: 468 N

3 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -2t + 2$, $y = 2t^2 + t - 2$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From the first equation: $t = \frac{x - 2}{-2}$

Substitute t into $y = 2t^2 + t - 2$ and expand.

$y = \frac{1}{2}x^2 - \frac{5}{2}x + 1$

Answer: $y = \frac{1}{2}x^2 - \frac{5}{2}x + 1$

4 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $3\sin^2 x + \sin x - 2 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

$(3 \sin x - 2)(\sin x + 1) = 0$, so $\sin x = \frac{2}{3}$ or $\sin x = -1$

$\sin x = \frac{2}{3}$: $x = 41.8°, 138.2°$

$\sin x = -1$: $x = 270.0°$

Answer: $x = 41.8°, 138.2°, 270.0°$

5 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5^{4x - 4} = 25^{-x - 5}$.

Write both sides as powers of 5: $5^{4x - 4} = 5^{2(-x - 5)}$
Equate the indices: $4x - 4 = -2x - 10$
$6x = -6$
$x = -1$
Answer: $x = -1$

6 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 12 pairs of values, $\bar{x} = 10$, $\bar{y} = 18$, $S_{xx} = 20$ and $S_{xy} = -10$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when x = 14.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-10}{20} = -0.5$

$a = \bar{y} - b\bar{x} = 18 - -0.5 \times 10 = 23$
$y = 23 - 0.5x$
(b) $y = 23 + (-0.5) \times 14 = 16$
Answer: $y = 23 - 0.5x$; $y = 16$

7 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is dropped from rest from a bridge, 11.025 m above the ground.
Take $g = 9.8$ m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $11.025 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 11.025 \div 4.9 = 2.25$, so $t = 1.5$
(b) $v = u + at = 9.8 \times 1.5$
Answer: (a) 1.5 s (b) 14.7 m s^{-1}

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{4x + 10}{(x + 1)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{4x + 10}{(x + 1)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{4x + 10}{(x + 1)(x + 4)} \equiv \frac{A}{x + 1} + \frac{B}{x + 4}$, so $4x + 10 \equiv A(x + 4) + B(x + 1)$
$x = -1$: $A = 2$; $x = -4$: $B = 2$
(b) $\int_0^4 \left(\frac{2}{x + 1} + \frac{2}{x + 4}\right) dx = [2\ln(x + 1) + 2\ln(x + 4)]_0^4$
$= 2\ln\frac{5}{1} + 2\ln\frac{8}{4} = \ln(100)$
Answer: (a) A = 2, B = 2 (b) ln(100)

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
$P(X < 32) = 0.1587$ and $P(X > 48) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$32 = \mu + (-0.9998)\sigma$ and $48 = \mu + 0.9998\sigma$
Subtract: $16 = 2.000\,\sigma$, so $\sigma = 8.00$
$\mu = 32 - (-0.9998)(8.001) = 40.0$
Answer: $\mu = 40.0$, $\sigma = 8.00$

10 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 10\,e^x + 24 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 10\,y + 24 = 0$
$(y - 6)(y - 4) = 0$, so $e^x = 6$ or $e^x = 4$
$x = \ln 4$ or $x = \ln 6$
Answer: $x = \ln 4$, $x = \ln 6$