Day 164 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 3t - 1.5t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 3 - 3t$
Maximum speed when $a = 0$: $t = 3 \div 3 = 1$
$v = 3 \times 1 - 1.5 \times 1^2$
Answer: 1.5 m s^{-1} at t = 1 s

2 Normal curve and shaded area (All boards)

$X \sim N(40, 10^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = P(30.00 < X < 40.00)
= P(X < 40.00) − P(X < 30.00)
= 0.5000 − 0.1587 = 0.341
Answer: 0.341

3 Trig equations in radians (All boards)

Solve $\sin(x + 0.5) = 0.8$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

Let $\theta = x + 0.5$, so $\sin\theta = 0.8$ for $0.5 \leq \theta \leq 2\pi + 0.5$
Calculator: $\theta = 0.9273$, then use the symmetry of the graph
$\theta = 0.9273, 2.214$
$x = \theta - 0.5$
$x = 0.4273, 1.714$
Answer: x = 0.427, 1.71

4 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A package is thrown horizontally at 29 m s^{-1} from the top of a building 78.4 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $78.4 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 16$
t = 4 s
Horizontal: $x = 29 \times 4$
Answer: 4 s, 116 m

5 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:

(a) $y = -f(x)$ (b) $y = f(\frac{x}{2})$

(a) $y = -f(x)$: A(−4, 1) maps to (−4, −1), B(−2, −2) maps to (−2, 2)

(b) $y = f(\frac{x}{2})$: A(−4, 1) maps to (−8, 1), B(−2, −2) maps to (−4, −2)

Answer: (a) (−4, −1), (−2, 2) (b) (−8, 1), (−4, −2)

6 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x^2 + 3$, the x-axis and the lines $x = 0$ and $x = 2$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^2 y^2 \, dx = \pi \int_0^2 (x^4 + 6x^2 + 9) \, dx$

$= \pi[\frac{x^5}{5} + 2x^3 + 9x]_0^2$

$= \frac{202\pi}{5}$

Answer: $\frac{202\pi}{5}$

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of $(3 + x)(1 + 2x)^4$.

$(1 + 2x)^4$ has x^2 coefficient 24 and x^3 coefficient 32
x^3 term of the product = 3 × 32 + 1 × 24
= 120
Answer: 120

8 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{4}} x \cos 2x \, dx$.

Let $u = x$ and $\frac{dv}{dx} = \cos 2x$, so $\frac{du}{dx} = 1$ and $v = \frac{\sin 2x}{2}$

Integral = $[uv] - \int v\frac{du}{dx} \, dx$

$= [\frac{x \sin 2x}{2}]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{2} \, dx$

$= \frac{\pi}{8} - \frac{1}{4}$

Answer: $\frac{\pi}{8} - \frac{1}{4}$

9 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 50 and s.d. 10.
A random sample of 4 is taken. Find the probability that the mean time is more than 54 (3 s.f.).

$\overline{X} \sim N(50, \frac{10^2}{4})$, standard deviation 5.000

$z = \frac{54 - 50}{5.000} = 0.8000$

P = 0.212
Answer: 0.212

10 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 12 that lie between 95 and 317.

First multiple = 12 × 8 = 96, last multiple = 12 × 26 = 312
This is an arithmetic series with d = 12 and 19 terms

$S = \frac{19}{2}(96 + 312) = 3876$

Answer: 3876