Day 163 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coach accelerates uniformly from rest at 3 m s^{-2} for 6 s, then travels at constant speed for 30 s, and finally decelerates uniformly at 0.5 m s^{-2} to rest.
Find the total distance travelled and the total time of the journey.

Max speed = 3 × 6 = 18
Stage 1: ½ × 6 × 18 = 54; stage 2: 18 × 30 = 540
Stage 3: time = 18 ÷ 0.5 = 36, distance = ½ × 18 × 36 = 324
Answer: 918 m; 72 s

2 Chain rule (All boards)

Given that $y = \sqrt{2x + 17}$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 4.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = \frac{1}{\sqrt{2x + 17}}$
When x = 4: gradient = $\frac{1}{5}$
Answer: $\frac{dy}{dx} = \frac{1}{\sqrt{2x + 17}}$; gradient $\frac{1}{5}$

3 Union, conditional probability and independence (All boards)

P(A) = 0.25, P(B) = 0.5, P(A ∩ B) = 0.25.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) P(A ∪ B) = 0.25 + 0.5 − 0.25 = 0.5
(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.5} = 0.5$
(c) P(A)P(B) = 0.25 × 0.5 = 0.125, which does not equal P(A ∩ B)
Answer: (a) 0.5 (b) 0.5 (c) not independent

4 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{6}{x - 4} - 1$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = 4, and y → −1 as x → ±∞
Asymptotes: x = 4 and y = −1
(b) x = 0: $y = -\frac{3}{2} - 1 = -\frac{5}{2}$
y = 0: $\frac{6}{x - 4} = 1$, so x − 4 = 6, x = 10
Answer: (a) x = 4, y = −1 (b) $(0, -\frac{5}{2})$ and (10, 0)

5 Rates of change (connected rates) (All boards)

The volume of a sphere increases at a constant rate of 12 $cm^3 s^{-1}$.
Find the rate at which the radius is increasing when r = 5 cm, to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$
$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{12}{4\pi r^2}$
When r = 5: $\frac{dr}{dt} = \frac{3}{25\pi} = 0.03820$
Answer: 0.0382 cm/s

6 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.5. It is tossed 20 times and lands heads 5 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.5.

H_0: p = 0.5, H_1: p ≠ 0.5. Under H_0, X ~ B(20, 0.5)
P(X ≤ 5) = 0.0207
Two-tailed: compare with 0.025. 0.0207 < 0.025, so reject H_0
There is sufficient evidence that the probability of heads is different from 0.5
Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.5

7 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 5)(x - 2)x \leq 0$.

Critical values: x = 0, x = 2, x = 5
The cubic has a positive x^3 term, so the graph goes from bottom left to top right
and crosses the axis at each critical value.
Reading from the sketch: x ≤ 0 or 2 ≤ x ≤ 5
Answer: x ≤ 0 or 2 ≤ x ≤ 5

8 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 - t^2 - 2t$ metres.
Find its velocity and acceleration when t = 3.

$v = ds/dt = 3t^2 - 2t - 2$
$a = dv/dt = 6t - 2$
t = 3: v = 19, a = 16
Answer: v = 19 m s^{-1}, a = 16 m s^{-2}

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \text{cosec}^2 x - 1$: $\text{cosec}^2 x - 1 = 0$
(cosec x − 1)(cosec x + 1) = 0, so cosec x = 1 or −1
cosec x = 1, so sin x = 1: x = 90.0°
cosec x = −1, so sin x = −1: x = 270.0°
Answer: x = 90.0°, 270.0°

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 3x + 4y = k is a tangent to the circle
$(x + 3)^2 + (y + 3)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
(−3, −3) ± 5 × (3, 4)/5 = (−3, −3) ± (3, 4)
T = (0, 1) or (−6, −7), and k = 3x + 4y gives:
k = 4 or k = −46
Answer: k = 4 or k = −46