Day 162 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^2)^2 \times 3x^3}{4x^3}$.

$(2x^2)^2 = 4x^4$

Numerator $= 4 \times 3x^7 = 12x^7$

Divide: $\frac{12}{4} = 3$ and $x^7 \div x^3 = x^4$

Answer: $3x^4$

2 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 9 pairs of data, $S_{xx} = 150$, $S_{yy} = 125$ and $S_{xy} = 58$.

Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{58}{\sqrt{150 \times 125}}$

$= \frac{58}{136.93} = 0.424$

r is positive, so there is weak positive correlation

Answer: r = 0.424, weak positive correlation

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 + 16x < -24$.

Rearrange: $2x^2 + 16x + 24 < 0$

Factorise: $2(x + 2)(x + 6) < 0$

Critical values: $x = -6$ and $x = -2$

Sketch the ∪-shaped parabola: $-6 < x < -2$

Answer: $-6 < x < -2$

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of height and arm span give $r = -0.683$.

One-tailed critical value (5%): 0.6215.

Test for negative correlation.

$H_0: \rho = 0$, $H_1: \rho < 0$

$r = -0.683$, critical value -0.6215; r is beyond the critical value in the right direction

r is in the critical region, so reject H_0

Conclusion: there is evidence of negative correlation between height and arm span

Answer: reject H_0: there is evidence of negative correlation between height and arm span

5 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 1.6 radians at the centre O of a circle of radius 10 cm.

Find the perimeter of the minor segment, to 3 s.f.

Arc $= r\theta = 10 \times 1.6 = 16$

Chord AB $= 2r \sin(\frac{\theta}{2}) = 14.35$

Perimeter $= 16 + 14.35 = 30.35$ cm

Answer: 30.3 cm

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 6| = |3x + 4|$.

Either $x - 6 = 3x + 4$ or $x - 6 = -(3x + 4)$

$x - 6 = 3x + 4$ gives $x = -5$

$x - 6 = -(3x + 4)$ gives $x = \frac{1}{2}$

Answer: $x = -5$ and $x = \frac{1}{2}$

7 Power, driving force and resistance (CIE)

A car of mass 1500 kg moves along a horizontal road. The engine works at a constant rate of 60 kW. The resistance to motion is constant at 600 N.

Find the acceleration when the speed is 20 m s^{-1}.

P = Fv: F = 60000 ÷ 20 = 3000 N

F − R = ma: 3000 − 600 = 1500 a

Answer: 1.6 m s^{-2}

8 Inequalities with exponents and logs (All boards)

£1000 is invested at 4% compound interest per year.

Find the least whole number of years for the investment to exceed £3000.

$1000 \times 1.04^n > 3000$, so $1.04^n > 3$

$n > \frac{\ln 3}{\ln 1.04} = 28.01$

The least whole number is n = 29

Answer: 29 years

9 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 + t)\mathbf{i} + (t^3)\mathbf{j}$ metres.

Find the velocity when t = 2, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t + 1)\mathbf{i} + (3t^2)\mathbf{j}$

t = 2: $\mathbf{v} = (9)\mathbf{i} + (12)\mathbf{j}$

Speed = $\sqrt{(9^2 + 12^2)}$

Answer: $\mathbf{v} = (9)\mathbf{i} + (12)\mathbf{j}$ m s^{-1}; speed 15 m s^{-1}

10 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -5 \\ -14 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 3.

Find the position vector of P.

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ -12 \end{pmatrix}$

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-5} \\ \mathbf{-5} \end{pmatrix}$