Day 161 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = e^x + 3$ to find the exact value of

$\int_0^{\ln 2} (e^x + 3)^2 e^x \, dx$.

$\frac{du}{dx} = e^x$, so $e^x \, dx = du$

Limits: $x = 0$ gives $u = 4$, $x = \ln 2$ gives $u = 5$

$\int_4^5 u^2 \, du = [\frac{u^3}{3}]_4^5$

$= \frac{61}{3}$

Answer: $\frac{61}{3}$

2 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $(-3\mathbf{i} - \mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-\mathbf{i})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-3\mathbf{i} - \mathbf{j}) + \mathbf{P} = 4(-\mathbf{i}) = (-4\mathbf{i})$

$\mathbf{P} = -\mathbf{i} + \mathbf{j}$

$|\mathbf{P}| = \sqrt{(-1^2 + 1^2)}$

Answer: $\mathbf{P} = (-\mathbf{i} + \mathbf{j})$ N; $|\mathbf{P}| = 1.41$ N

3 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 12x + 6y + 18 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x - 6)^2 - 36 + (y + 3)^2 - 9 + 18 = 0$

$(x - 6)^2 + (y + 3)^2 = 27$

Centre $(6, -3)$, radius $3\sqrt{3}$

Answer: centre $(6, -3)$, radius $3\sqrt{3}$

4 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 4 \\ -4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \\ r \end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\text{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} -2 \\ 1 \\ r - -2 \end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 1 gives $-2 = \lambda \times 2$, so $\lambda = -1$

$r - -2 = -1 \times 3$, so $r = -5$

Answer: $r = -5$

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5 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point $(-2, 7)$ and B is the point $(4, -5)$.
Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = (1, 1)

Gradient of AB = $\frac{-12}{6} = -2$

Perpendicular gradient = $\frac{1}{2}$

$y - 1 = \frac{1}{2}(x - 1)$, which gives $x - 2y + 1 = 0$

Answer: $x - 2y + 1 = 0$

6 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 5g = 5a$
Hanging: $5g - T = 5a$
Add: $5g - 1g = 10a$
$a = 3.92$
$T = 5(9.8 - 3.92)$

Answer: $a = 3.92$ m s^{-2}; $T = 29.4$ N

7 Two independent binomials (All boards)

$X \sim B(6, 0.2)$ and $Y \sim B(8, 0.1)$ are independent.
Find $P(X = 1 \text{ and } Y \leq 2)$, to 3 significant figures.

$P(X = 1) = 0.3932$
$P(Y \leq 2) = 0.9619$
Independent, so multiply: $0.3932 \times 0.9619 = 0.378$

Answer: 0.378

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{4}{(x + 3)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{4}{(x + 3)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{4}{(x + 3)(x + 5)} \equiv \frac{A}{x + 3} + \frac{B}{x + 5}$, so $4 \equiv A(x + 5) + B(x + 3)$
$x = -3$: $A = 2$; $x = -5$: $B = -2$

(b) $\int_0^3 \left(\frac{2}{x + 3} + \frac{-2}{x + 5}\right) dx = [2 \ln(x + 3) - 2 \ln(x + 5)]_0^3$

$= 2 \ln \frac{6}{3} - 2 \ln \frac{8}{5} = \ln\left(\frac{25}{16}\right)$

Answer: (a) $A = 2$, $B = -2$ (b) $\ln\left(\frac{25}{16}\right)$

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9 Continuous random variable: median (CIE)

The continuous random variable X has pdf f(x) = $\frac{2x}{100}$ for $0 \leq x \leq 10$, and 0 otherwise.

(a) Find P(X > 1).

(b) Find the median of X, to 3 significant figures.

F(x) = $\int_0^x$ f(t) dt = $\frac{x^2}{100}$

(a) P(X > 1) = 1 − F(1) = $\frac{99}{100}$

(b) F(m) = 0.5: $\frac{m^2}{100} = \frac{1}{2}$

m = 7.07

Answer: (a) $\frac{99}{100}$ (b) 7.07

10 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{9}{4})^{-\frac{1}{2}}$.

$\frac{9}{4} = (\frac{3}{2})^2$, so the expression = $(\frac{3}{2})^{-1}$

$= \frac{2}{3}$

Answer: $\frac{2}{3}$