Day 160 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 24 kg is hinged at A and held horizontal by a light string attached at B. The string makes an angle θ with the rod, where $\sin\theta = \frac{4}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 3 = mg \times 1.5$

$T \times \frac{4}{5} \times 3 = 235.2 \times 1.5$

$T = 352.8 \div 2.4$

Answer: 147 N

2 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x - 6| < -3x - 4$.

Critical values: $-x - 6 = -3x - 4$ gives $x = 1$; $-x - 6 = -(-3x - 4)$ gives $x = -\frac{5}{2}$

Sketch $y = |-x - 6|$ and $y = -3x - 4$, and compare the graphs at $x = 1$ and $x = -\frac{5}{2}$.

$|-x - 6| < -3x - 4$ when $x < -\frac{5}{2}$

Answer: $x < -\frac{5}{2}$

3 Conditional probability from a count (All boards)

80 customers: 21 buy coffee, 36 buy tea, 30 do neither.
(a) How many do both?
(b) Given a person who buys coffee, find P(this person buys tea).

(a) At least one: $80 - 30 = 50$. Both $= 21 + 36 - 50 = 7$

(b) Of the 21 who buy coffee, 7 also do the other: $P = \frac{7}{21} = \frac{1}{3}$

Answer: (a) 7 (b) $\frac{1}{3}$

4 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 7 cm and angle 60°.
Find the perimeter of the sector, in terms of π.

$\theta = \frac{\pi}{3}$ radians

Arc length $= r\theta = \frac{7\pi}{3}$ cm

Perimeter $= 2r + \text{arc} = \frac{7\pi}{3} + 14$ cm

Answer: $\frac{7\pi}{3} + 14$ cm

5 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £15000. Its value falls by 15% each year.
(a) Find its value after 4 years (nearest £).
(b) In which year does it first fall below £4000?

(a) Value $= 15000 \times 0.85^4 =$ £7,830

(b) $15000 \times 0.85^n < 4000$, so $n \lg 0.85 < \lg(4000 \div 15000)$
$n > \lg(4000 \div 15000) \div \lg 0.85 = 8.133$ (dividing by a negative number)
So $n = 9$

Answer: (a) £7,830 (b) 9 years

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6 Finding the mean or standard deviation (All boards)

A battery lasts X hours, normal with mean 50.
Given P(X < 52.5) = 0.8, find the standard deviation (3 s.f.).

$z = 0.8416$, so $\frac{52.5 - 50}{\sigma} = 0.8416$

$\sigma = 2.5 \div 0.8416 = 2.97$

Answer: $\sigma = 2.97$

7 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{u_n - 1}{u_n + 1}$, $u_1 = \frac{1}{2}$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{25} and $\sum_{r=1}^{25} u_r$.

(a) $u_2 = -\frac{1}{3}$, $u_3 = -2$, $u_4 = 3$, $u_5 = \frac{1}{2}$

$u_5 = u_1$ so the sequence repeats with period 4

(b) $25 = 4 \times 6 + 1$, so $u_{25} = u_1 = \frac{1}{2}$

Sum of one cycle $= \frac{7}{6}$, so the sum $= 6 \times \frac{7}{6} + \frac{1}{2} = \frac{15}{2}$

Answer: period 4; $u_{25} = \frac{1}{2}$; sum $= \frac{15}{2}$

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (1 kg) and B (9 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 2 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (9 - 1)g \div (9 + 1) = 7.84$

$v^2 = 0 + 2 \times 7.84 \times 2 = 31.36$

$v = 5.6$

Answer: 5.6 m s^{-1}

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan^2 x - 4 \sec x + 4 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $\sec^2 x - 4 \sec x + 3 = 0$

$(\sec x - 3)(\sec x - 1) = 0$, so $\sec x = 3$ or 1

$\sec x = 3$, so $\cos x = \frac{1}{3}$: $x = 70.5°, 289.5°$

$\sec x = 1$, so $\cos x = 1$: $x =$

Answer: $x = 70.5°, 289.5°$

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10 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution u = cos x to find the exact value of

$\int_0^{\frac{\pi}{3}} \sin x \cos^4 x \, dx$.

$\frac{du}{dx} = -\sin x$, so $\sin x \, dx = -du$

Limits: x = 0 gives u = 1, x = $\frac{\pi}{3}$ gives u = $\frac{1}{2}$

$\int_1^{\frac{1}{2}} -u^4 \, du = [-\frac{u^5}{5}]_1^{\frac{1}{2}}$

$= \frac{31}{160}$

Answer: $\mathbf{\frac{31}{160}}$