Day 16 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-4, 6)$ and $(-1, -1)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-4 + (-1)}{2}, \frac{6 + (-1)}{2}\right) = \left(-\frac{5}{2}, \frac{5}{2}\right)$

(b) Gradient $= \frac{-7}{3} = -\frac{7}{3}$

$y - 6 = -\frac{7}{3}(x + 4)$

Multiply out and collect terms: $7x + 3y + 10 = 0$

Answer: (a) $\left(-\frac{5}{2}, \frac{5}{2}\right)$ (b) $7x + 3y + 10 = 0$

2 Basic trig equations (degrees) (All boards)

Solve $\cos x = \frac{\sqrt{2}}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = 45°$
Second solution: $360° - 45° = 315°$
Answer: $x = 45°, 315°$

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 8 m s^{-1} brakes and comes to rest with constant deceleration 4 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 8^2 - 2 \times 4 \times s$
$s = 64 \div 8 = 8$
(b) $0 = 8 - 4t$, so $t = 8 \div 4$
Answer: (a) 8 m (b) 2 s

4 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £18000. Its value falls by 15% each year.
(a) Find its value after 6 years (nearest £).
(b) In which year does it first fall below £5000?

(a) Value $= 18000 \times 0.85^6 =$ £6,789
(b) $18000 \times 0.85^n < 5000$, so $n \lg 0.85 < \lg(5000 \div 18000)$
$n > \lg(5000 \div 18000) \div \lg 0.85 = 7.882$ (dividing by a negative number)
So $n = 8$
Answer: (a) £6,789 (b) 8 years

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 3x - 7$. Use the Newton-Raphson method with
$x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{-3.000}{6.000} = 1.5000$

$x_2 = 1.5000 - \frac{0.8750}{9.750} = 1.4103$

Answer: $x_1 = 1.5000$, $x_2 = 1.4103$

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (7, −1) and B is the point (1, 6).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(4, \frac{5}{2})$

Gradient of AB = $\frac{7}{-6} = -\frac{7}{6}$

Perpendicular gradient = $\frac{6}{7}$

$y - \frac{5}{2} = \frac{6}{7}(x - 4)$, which gives $12x - 14y - 13 = 0$

Answer: $12x - 14y - 13 = 0$

7 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 150 and s.d. 8.
A random sample of 16 is taken. Find the probability that the mean time is more than 154 (3 s.f.).

$\overline{X} \sim N(150, \frac{8^2}{16})$, standard deviation 2.000

$z = \frac{154 - 150}{2.000} = 2.000$

P = 0.0228

Answer: 0.0228

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (2 kg) and B (3 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 4.5 m above the ground, then released.
Take g = 9.8 m s^{-2}.
Find the speed with which B hits the ground.

$a = (3 - 2)g \div (3 + 2) = 1.96$

$v^2 = 0 + 2 \times 1.96 \times 4.5 = 17.64$

v = 4.2

Answer: 4.2 m s^{-1}

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cot^2 x - 7\operatorname{cosec} x + 8 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $2\operatorname{cosec}^2 x - 7\operatorname{cosec} x + 6 = 0$

$(\operatorname{cosec} x - 2)(2\operatorname{cosec} x - 3) = 0$, so $\operatorname{cosec} x = 2$ or $\frac{3}{2}$

$\operatorname{cosec} x = 2$, so $\sin x = \frac{1}{2}$: x = 30.0°, 150.0°

$\operatorname{cosec} x = \frac{3}{2}$, so $\sin x = \frac{2}{3}$: x = 41.8°, 138.2°

Answer: x = 30.0°, 41.8°, 138.2°, 150.0°

10 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 32 measurements of m.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: Σfx = 1530, so mean = 1530 ÷ 32 = 47.8

(b) $\frac{n}{2}$ = 16. Cumulative frequency before the median class = 13

Median class 50 to 60. Median = $50 + \frac{3}{13} \times 10 = 52.3$

Answer: mean ≈ 47.8, median ≈ 52.3