Day 159 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{1}{x-3} - 3$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = 3$, and $y \to -3$ as $x \to \pm\infty$

Asymptotes: $x = 3$ and $y = -3$

(b) $x = 0$: $y = -\frac{1}{3} - 3 = -\frac{10}{3}$

$y = 0$: $\frac{1}{x-3} = 3$, so $x - 3 = \frac{1}{3}$, $x = \frac{10}{3}$

Answer: (a) $x = 3$, $y = -3$ (b) $(0, -\frac{10}{3})$ and $(\frac{10}{3}, 0)$

2 Trig equations in radians (All boards)

Solve $\sin x = 0.5$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

$\sin\theta = 0.5$ for $0 \leq x \leq 2\pi$

Calculator: $x = 0.5236$, then use the symmetry of the graph

$x = 0.5236, 2.618$

Answer: x = 0.524, 2.62

3 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 5)(x + 2)(x + 4) > 0$.

Critical values: $x = -5$, $x = -4$, $x = -2$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right
and crosses the axis at each critical value.

Reading from the sketch: $-5 < x < -4$ or $x > -2$

Answer: $-5 < x < -4$ or $x > -2$

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3\sec 2x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{8}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 6\tan 2x \sec 2x$

At $x = \frac{\pi}{8}$: $\frac{dy}{dx} = 6\sqrt{2}$

Answer: $\frac{dy}{dx}$ = 6 tan 2x sec 2x; gradient $6\sqrt{2}$

5 Binomial mean and variance (All boards)

X has a binomial distribution with mean 4.5 and variance 3.15.

(a) Find n and p.

(b) Find P(X = 4) to 3 significant figures.

(a) $np = 4.5$ and $np(1 - p) = 3.15$

$1 - p = 3.15 \div 4.5 = 0.7$, so $p = 0.3$ and $n = 4.5 \div 0.3 = 15$

(b) $P(X = 4) = {}^{15}C_4 \times 0.3^4 \times 0.7^{11} = 0.219$

Answer: (a) n = 15, p = 0.3 (b) 0.219

6 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A golf ball is projected from ground level with initial velocity components 32 m s^{-1} horizontally and 9.8 m s^{-1} vertically upwards.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 9.8^2 - 2 \times 9.8 \times H$, $H = 96.04 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 4.9 m (b) 32 m s^{-1}

7 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term a and common ratio r, where $|r| < 1$.

The sum to infinity is $\frac{25}{4}$ times the second term.

Find the possible values of r.

$\frac{a}{1-r} = \frac{25}{4} \times ar$, so $1 = \frac{25}{4} r(1-r)$

$\frac{25}{4}r^2 - \frac{25}{4}r + 1 = 0$

Multiply by 4: $25r^2 - 25r + 4 = 0$

$r = \frac{1}{5}$ or $r = \frac{4}{5}$ (both satisfy $|r| < 1$)

Answer: $r = \frac{1}{5}$ or $r = \frac{4}{5}$

8 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.25, find the least number of traps needed for the probability of at least one success to exceed 0.95.

P(at least one) $= 1 - 0.75^n > 0.95$

$0.75^n < 0.05$

$n > \frac{\log 0.05}{\log 0.75} = 10.41$

Answer: n = 11

9 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 12 kg rests on a rough horizontal floor, with coefficient of friction 0.25. A horizontal force of 35 N is applied to the box.

Take g = 9.8 m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

R = 12 × 9.8 = 117.6, maximum friction $\mu R = 0.25 \times 117.6 = 29.4$

35 > 29.4 so the box moves, F = 29.4

$35 - 29.4 = 12\ a$ (3 s.f.)

Answer: It moves; a = 0.467 m s^{-2}

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10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-4x-16}{(x+1)(x+5)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{-4x-16}{(x+1)(x+5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-4x-16}{(x+1)(x+5)} \equiv \frac{A}{x+1} + \frac{B}{x+5}$, so $-4x-16 \equiv A(x+5) + B(x+1)$
$x = -1$: $A = -3$; $x = -5$: $B = -1$

(b) $\int_0^2 \left(\frac{-3}{x+1} + \frac{-1}{x+5}\right) dx = [-3\ln(x+1) - \ln(x+5)]_0^2$
$= -3\ln\frac{3}{1} - \ln\frac{7}{5} = \ln\left(\frac{5}{189}\right)$

Answer: (a) $A = -3$, $B = -1$ (b) $\ln\left(\frac{5}{189}\right)$