Day 158 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 6\times2^{x} + 8 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 - 6y + 8 = 0$

$(y - 2)(y - 4) = 0$

$y = 2$ or $y = 4$

$2^x = 2$ gives $x = 1$; $2^x = 4$ gives $x = 2$.

Answer: $x = 1$ or $x = 2$

2 Binomial probabilities (All boards)

$X \sim B(19, 0.4)$. Find, to 3 significant figures,

(a) $P(X > 8)$

(b) $P(X = 8)$

(a) $P(X > 8) = 1 - P(X \leq 8)$

(a) = 0.333

(b) $P(X = 8) = {}^{19}C_8 \times 0.4^8 \times 0.6^{11} = 0.180$

Answer: (a) 0.333 (b) 0.180

3 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 5 m and mass 30 kg rests horizontally on supports at C and D, where AC = 1.5 m and BD = 0.5 m.

A particle of mass 10 kg is placed on the beam 2 m from B. Take $g = 9.8$ m s^{-2}.

Find the reactions at C and D.

Moments about C: $R_D \times 3 = 30\ g \times 1 + 10\ g \times 1.5$

$R_D = 441 \div 3$

Resolve vertically: $R_C + R_D = 40\ g$

Answer: $R_C = 245$ N; $R_D = 147$ N

4 Solving exponential equations using logarithms (All boards)

Solve $e^{2x} = 50$, giving your answer to 3 s.f.

Take natural logs: $2x = \ln 50$

$x = \frac{\ln 50}{2} = 1.956$

Answer: $x = 1.96$

5 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 2x + 8y - 10 = 0$.

Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x - 1)^2 - 1 + (y + 4)^2 - 16 - 10 = 0$

$(x - 1)^2 + (y + 4)^2 = 27$

Centre $(1, -4)$, radius $3\sqrt{3}$

Answer: centre $(1, -4)$, radius $3\sqrt{3}$

6 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 18t - 3t^2$ m s^{-1}, for $t \geq 0$.

Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 18 - 6t$

Maximum speed when $a = 0$: $t = 18 \div 6 = 3$

$v = 18 \times 3 - 3 \times 3^2$

Answer: 27 m s^{-1} at t = 3 s

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7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 24 cos θ + 7 sin θ ≡ 25 cos(θ − 16.3°),
solve 24 cos θ + 7 sin θ = −4 for 0 ≤ θ ≤ 360°. (1 d.p.)

25 cos(θ − 16.3°) = −4, so cos(θ − 16.3°) = $\frac{-4}{25}$

θ − 16.3° = ±99.2°
θ = 16.3° + 99.2° = 115.5°
θ = 16.3° − 99.2° = −82.9°, so θ = 277.1°
Answer: θ = 115.5°, 277.1°

8 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 18, is a bar 2 cm wide, 2.25 cm tall.
(a) Find the bar height for width 30, frequency 45.
(b) Find the frequency of a 1.5 cm by 2 cm bar.

Area of bar ∝ frequency: 2 × 2.25 = 4.5 cm² represents 18
so 1 unit of frequency = 0.25 cm²
(a) Width = 6 cm, area = 0.25 × 45 = 11.25 cm², height = 11.25 ÷ 6 = 1.875 cm
(b) Area = 1.5 × 2 = 3 cm², frequency = 3 ÷ 0.25 = 12
Answer: (a) 1.875 cm (b) 12

9 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(2 + 2\sqrt{6})(5 - \sqrt{6})$, giving your answer in the form $a + b\sqrt{6}$.

$(2 + 2\sqrt{6})(5 - \sqrt{6}) = 10 - 2\sqrt{6} + 10\sqrt{6} - 2\sqrt{6}\times\sqrt{6}$
$\sqrt{6}\times\sqrt{6} = 6$, so the expression $= -2 + 8\sqrt{6}$
Answer: $-2 + 8\sqrt{6}$

10 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $10\cos^2 x - 3\cos x - 1 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(2\cos x - 1)(5\cos x + 1) = 0$, so $\cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{5}$

$\cos x = \frac{1}{2}$: x = 60.0°, 300.0°

$\cos x = -\frac{1}{5}$: x = 101.5°, 258.5°

Answer: x = 60.0°, 101.5°, 258.5°, 300.0°