**Day 157** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 1}$.

$\frac{dy}{dx} = \frac{1 - x^2}{(x^2 + 1)^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $-(x - 1)(x + 1) = 0$

$x = -1$ or $x = 1$

**Answer: $x = -1, x = 1$**

## 2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 8x - 4y + 2 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x - 4)^2 - 16 + (y - 2)^2 - 4 + 2 = 0$

$(x - 4)^2 + (y - 2)^2 = 18$

Centre (4, 2), radius $3\sqrt{2}$

**Answer: centre (4, 2), radius $3\sqrt{2}$**

## 3 Trig equations in radians (All boards)

Solve $\sin(x + 0.3) = -0.1$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

Let $\theta = x + 0.3$, so $\sin\theta = -0.1$ for $0.3 \leq \theta \leq 2\pi + 0.3$

Calculator: $\theta = -0.1002$, then use the symmetry of the graph

$\theta = 3.242, 6.183$

$x = \theta - 0.3$

$x = 2.942, 5.883$

**Answer: $x = 2.94, 5.88$**

## 4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$x^3 - x^2 - 5x + 5$ is divided by $(x - 2)$.

$x^2 \times (x - 2) = x^3 - 2x^2$, leaving $x^2 - 5x + 5$

$x \times (x - 2) = x^2 - 2x$, leaving $-3x + 5$

$-3 \times (x - 2) = -3x + 6$, leaving $-1$

Quotient $x^2 + x - 3$, remainder $-1$

**Answer: quotient $x^2 + x - 3$, remainder $-1$**

## 5 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $(3\mathbf{i} - 5\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(2\mathbf{i} - 2\mathbf{j})$ m s$^{-2}$.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

$\mathbf{F} = m\mathbf{a}$: $(3\mathbf{i} - 5\mathbf{j}) + \mathbf{P} = 5(2\mathbf{i} - 2\mathbf{j}) = (10\mathbf{i} - 10\mathbf{j})$

$\mathbf{P} = 7\mathbf{i} - 5\mathbf{j}$

$|\mathbf{P}| = \sqrt{(7^2 + -5^2)}$

**Answer: $\mathbf{P} = (7\mathbf{i} - 5\mathbf{j})$ N; $|\mathbf{P}| = 8.60$ N**

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## 6 Binomial critical region (All boards)

X ~ B(20, p). $H_0$: p = 0.4 is to be tested against $H_1$: p < 0.4 at the 5% significance level.
Find the critical region and the actual significance level.

Under $H_0$, X ~ B(20, 0.4)
$P(X \leq 3) = 0.0160 \leq 0.05$
$P(X \leq 4) = 0.0510 > 0.05$
The critical region is $X \leq 3$
**Answer: $X \leq 3$; actual significance level 1.60%**

## 7 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = t^2$, $y = t^3$.
Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at $t = -1$.

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2$
$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t}{2}$; at $t = -1$ it is $-\frac{3}{2}$
Point: $(1, -1)$
$y + 1 = -\frac{3}{2}(x - 1)$, so $y = -\frac{3}{2}x + \frac{1}{2}$
**Answer: $\frac{dy}{dx} = \frac{3t}{2}$; tangent $y = -\frac{3}{2}x + \frac{1}{2}$**

## 8 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 7 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.
Take g = 9.8 m $s^{-2}$.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$
Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 68.6$
From the first: $T_1 = \frac{3}{4} T_2$; into the second: $T_2 \times \frac{5}{4} = 68.6$
**Answer: $T_1$ = 41.16 N (at α); $T_2$ = 54.88 N (at β)**

## 9 Finding the mean or standard deviation (All boards)

The length X cm of a metal rod is normal with s.d. 4.
Given P(X < 59.3) = 0.99, find the mean (3 s.f.).

P(X < 59.3) = 0.99, so z = 2.326
$\frac{59.3 - \mu}{4} = 2.326$
$\mu = 59.3 - 2.326 \times 4 = 50.0$
**Answer: $\mu$ = 50.0**

## 10 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-7x + 15}{(2x - 3)(x - 3)}$ in partial fractions.

$\frac{-7x + 15}{(2x - 3)(x - 3)} \equiv \frac{A}{(2x - 3)} + \frac{B}{(x - 3)}$, so $-7x + 15 \equiv A(x - 3) + B(2x - 3)$
$x = \frac{3}{2}$: $\frac{9}{2} = A \times (-\frac{3}{2})$, so A = −3
x = 3: −6 = B × 3, so B = −2
**Answer: $-\frac{3}{(2x - 3)} - \frac{2}{(x - 3)}$**