Day 156 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^4 (x^2 + \frac{6}{x})\,dx$.

$\int (x^2 + \frac{6}{x})\,dx = \frac{x^3}{3} + 6\ln x$

$= [\frac{x^3}{3} + 6\ln x]_1^4$

$= \ln 4096 + 21$

Answer: $\ln 4096 + 21$

2 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is $(6, -3)$ and B is $(-2, -5)$.
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = $(2, -4)$

Radius$^2 = (6 - 2)^2 + (-3 - (-4))^2 = 17$

Answer: $(x - 2)^2 + (y + 4)^2 = 17$

3 Probability without replacement (All boards)

A bag has 6 green and 9 yellow beads; two are taken without replacement.
(a) Find P(exactly one green).
(b) Find P(at least one green).

(a) P(green then yellow) + P(yellow then green) $= 2 \times \frac{6}{15} \times \frac{9}{14} = \frac{18}{35}$

(b) $1 -$ P(both yellow) $= 1 - \frac{9}{15} \times \frac{8}{14} = 1 - \frac{12}{35} = \frac{23}{35}$

Answer: (a) $\frac{18}{35}$ (b) $\frac{23}{35}$

4 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve $y = f(x)$, where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

The curve touches the x-axis at $x = 4$ (repeated root) and crosses at $x = -1$

So $y = k(x - 4)^2(x + 1)$

The curve passes through $(0, -16)$: $k \times 16 \times 1 = -16$

So $k = -1$

Answer: $f(x) = -(x - 4)^2(x + 1)$

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5 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 14 \\ 4 \\ 17 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 3.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} 12 \\ 8 \\ 12 \end{pmatrix}$

$\text{AP} = \frac{1}{4}\text{AB} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 5 \\ -2 \\ 8 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{5} \\ \mathbf{-2} \\ \mathbf{8} \end{pmatrix}$

6 Total probability and Bayes (All boards)

R: made by machine A; L: item is faulty.

P(R) = 0.6, P(L | R) = 0.5, P(L | R') = 0.1.

(a) Find P(L).

(b) Find P(R | L).

(a) $\text{P(L)} = 0.6 \times 0.5 + 0.4 \times 0.1 = 0.3 + 0.04 = 0.34$

(b) $\text{P(R | L)} = \frac{0.6 \times 0.5}{0.34} = \frac{0.3}{0.34} = 0.882$

Answer: (a) 0.34 (b) $\mathbf{\frac{15}{17}}$

7 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\text{v} = (2t^2 + 4)\mathbf{i} + (4t - 16)\mathbf{j}$ m s^{-1}.

Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0

$4t - 16 = 0$, so $t = 4$

$\text{v} = (2 \times 4^2 + 4)\mathbf{i} = 36\mathbf{i}$

Answer: t = 4 s, speed 36 m s^{-1}

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{5}$ is irrational, prove by contradiction that $6 + 6\sqrt{5}$ is irrational.

Assume $6 + 6\sqrt{5} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $6\sqrt{5} = \frac{r}{s} - 6 = \frac{r - 6s}{s}$, so $\sqrt{5} = \frac{r - 6s}{6s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{5}$ being irrational, so $6 + 6\sqrt{5}$ is irrational.

Answer: $6 + 6\sqrt{5}$ is irrational

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -6 \\ -3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-6 + 3\lambda = 6 + (-2)\mu$

j: $-3 + 1\lambda = 5 + (-2)\mu$

Solving i and j: $\lambda = 2$, $\mu = 3$

k check: $-1 + 0(2) = -1$ and $-7 + 2(3) = -1$, equal

Point: $\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \\ \mathbf{-1} \end{pmatrix}$

10 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 65 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 15 to t = 55.

The floors are 525 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (65 + 40) × V = 525

105 V = 1050 so V = 10

Deceleration = 10 ÷ 10

Answer: V = 10; deceleration 1 m s^{-2}