Day 155 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + 5n + 5$ is odd.

Let n = 2k + 1, where k is an integer.

$n^2 = 4k^2 + 4k + 1$

$5n = 10k + 5$

Total = $4k^2 + 14k + 11$

4k + 14 is even and 11 is odd, so the total is odd.

Answer: the expression is odd

2 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cot 315°,
giving your answer with a rational denominator.

$\cot 315° = \frac{1}{\tan 315°}$

$\tan 315° = -1$, so $\cot 315° = \frac{1}{-1}$

Answer: -1

3 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (t^2 + t)\mathbf{i} + (t^3 - 3t)\mathbf{j}$ metres.
Find the velocity when t = 3, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (2t + 1)\mathbf{i} + (3t^2 - 3)\mathbf{j}$

t = 3: $\mathbf{v} = (7)\mathbf{i} + (24)\mathbf{j}$

Speed = $\sqrt{(7^2 + 24^2)}$

Answer: $\mathbf{v} = (7)\mathbf{i} + (24)\mathbf{j}$ m s^{-1}; speed 25 m s^{-1}

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 15 pairs of data, $S_{xx} = 50$, $S_{yy} = 125$ and $S_{xy} = 74$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{74}{\sqrt{50 \times 125}}$

$= \frac{74}{79.057} = 0.936$

r is positive, so there is strong positive correlation

Answer: r = 0.936, strong positive correlation

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arctan(\tan \frac{3\pi}{4})$.

$\tan \frac{3\pi}{4} = -1$

$\arctan(-1)$ must lie in $(-\frac{\pi}{2}, \frac{\pi}{2})$, so the value is $-\frac{\pi}{4}$

Answer: $-\frac{\pi}{4}$

6 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 9k + 6, where k is an integer.

Write the integer as n = 9q + j, where j = 0, 1, ..., 8.

n^3 leaves the same remainder as j^3 when divided by 9.

j = 0, 1, 2, 3, 4, 5, 6, 7, 8 give remainders 0, 1, 8, 0, 1, 8, 0, 1, 8.

The only possible remainders are 0, 1, 8, never 6.

Answer: possible remainders 0, 1, 8, never 6

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7 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 3t + 3$ m s^{-1}.
Find the distance moved between t = 0 and t = 2. (The particle does not change direction.)

$s = \int_0^2 (3t^2 - 3t + 3)\, dt$

$= [\, t^3 - \frac{3}{2}t^2 + 3t \,]$ from t = 0 to t = 2

$= 8 - 0$

Answer: 8 m

8 Binomial two-tailed test (All boards)

A factory says a proportion 0.5 of its parts are defective. In a sample of 30 parts, 12 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.5.

H_0: p = 0.5, H_1: $p \neq 0.5$. Under H_0, X ~ B(30, 0.5)

$P(X \leq 12) = 0.1808$

Two-tailed: compare with 0.025. 0.1808 > 0.025, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.5

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.5

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $4x - 3y = k$ is a tangent to the circle
$(x - 4)^2 + (y - 2)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(4, -3)$.
A unit step in this direction is $(4, -3)/5$, so the points of contact are
$(4, 2) \pm 5 \times (4, -3)/5 = (4, 2) \pm (4, -3)$
T = $(8, -1)$ or $(0, 5)$, and $k = 4x - 3y$ gives:
k = 35 or k = −15

Answer: k = 35 or k = −15

10 Basic trig equations (degrees) (All boards)

Solve $\tan(x + 50°) = 4$ for $0 \leq x \leq 360°$, giving answers to 1 d.p.

Let $\theta = x + 50°$, so $\tan\theta = 4$ with θ in range 50° to 410°
$\theta = 75.96°, 255.96°$
x = 26.0°, 206.0°

Answer: x = 26.0°, 206.0°