Day 154 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (3x^3 + 1 - \frac{2}{x^2})\,dx$.

$\int (3x^3 + 1 - \frac{2}{x^2})\,dx = \frac{3x^4}{4} + x + \frac{2}{x}$

$= [\frac{3x^4}{4} + x + \frac{2}{x}]_1^2$

$= \frac{45}{4}$

Answer: $\frac{45}{4}$

2 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + n + 11$ is odd for all integers n.

$n^2 + n = n(n + 1)$

As 1 is odd, n and n + 1 have opposite parity,

so one of them is even and the product n(n + 1) is even.

Even + 11 = odd, so $n^2 + n + 11$ is odd.

Answer: n(n + 1) is even, so the expression is odd

3 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.

Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 12 ÷ 4

(b) Distance = area of trapezium = ½ × (52 + 40) × 12

Answer: (a) 3 m s^{-2} (b) 552 m

4 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 - 6x + 1$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 3x^2 - 6x + 1$.

Take out the factor 3: $3x^2 - 6x + 1 = 3(x^2 - 2x) + 1$

$= 3[(x - 1)^2 - 1] + 1$

$= 3(x - 1)^2 - 2$

$(x - 1)^2 \geq 0$ with equality at x = 1, so the minimum is −2 at x = 1.

Answer: (a) $3(x - 1)^2 - 2$ (b) (1, −2)

5 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{6}} x \cos 3x\,dx$.

Let u = x and $\frac{dv}{dx} = \cos 3x$, so $\frac{du}{dx} = 1$ and $v = \frac{\sin 3x}{3}$

Integral = $[uv] - \int v\frac{du}{dx}\,dx$

$= [\frac{x \sin 3x}{3}]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \frac{\sin 3x}{3}\,dx$

$= \frac{\pi}{18} - \frac{1}{9}$

Answer: $\frac{\pi}{18} - \frac{1}{9}$

6 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1-u_n}$, $u_1 = 3$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{40} and $\sum_{r=1}^{40} u_r$.

(a) $u_2 = -\frac{1}{2}$, $u_3 = \frac{2}{3}$, $u_4 = 3$

$u_4 = u_1$ so the sequence repeats with period 3

(b) $40 = 3 \times 13 + 1$, so $u_{40} = u_1 = 3$

Sum of one cycle = $\frac{19}{6}$, so the sum = $13 \times \frac{19}{6} + 3 = \frac{265}{6}$

Answer: period 3; $u_{40} = 3$; sum = $\frac{265}{6}$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 8 g. Claimed mean 50 g; 25 packets have mean 53.4 g.
Test at the 5% level whether the true mean is greater than 50.

$H_0: \mu = 50$, $H_1: \mu > 50$. Under H_0, $\overline{X} \sim N(50, \frac{8^2}{25})$

$z = \frac{53.4 - 50}{8/\sqrt{25}} = 2.125$

$P(Z > 2.125) = 0.0168 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean

8 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $5\cos^2 x - \sin x - 5 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $-5\sin^2 x - \sin x = 0$, i.e. $5\sin^2 x + \sin x = 0$

$(\sin x)(5\sin x + 1) = 0$, so $\sin x = 0$ or $-\frac{1}{5}$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$

$\sin x = -\frac{1}{5}$: $x = 191.5°, 348.5°$

Answer: $x = 0.0°, 180.0°, 191.5°, 348.5°, 360.0°$

9 Inverse normal (All boards)

A battery lasts X hours, normal with mean 60, s.d. 4.
Find k (3 s.f.): (a) $P(X < k) = 0.05$,
(b) $P(X > k) = 0.1$.

(a) $z = -1.645$, so $k = 60 + (-1.645) \times 4 = 53.4$

(b) $P(X < k) = 0.9$, $z = 1.282$, so $k = 60 + 1.282 \times 4 = 65.1$

Answer: (a) 53.4 (b) 65.1

10 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(4\mathbf{i} + 5\mathbf{j})$ N and $(8\mathbf{i})$ N act on a particle of mass 2 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(4\mathbf{i} + 5\mathbf{j}) + (8\mathbf{i}) = 12\mathbf{i} + 5\mathbf{j}$

$|R| = \sqrt{(12^2 + 5^2)} = 13$ N

$a = F/m = 13 \div 2$

Answer: 6.5 m s^{-2}