Day 153 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A sports club has 64 junior members, 120 senior members and 40 veteran members.
A stratified sample of size 28 is taken. Find the number selected from each group.

Total = 64 + 120 + 40 = 224

Sampling fraction = 28/224 = $\frac{1}{8}$

junior: 64 ÷ 8 = 8
senior: 120 ÷ 8 = 15
veteran: 40 ÷ 8 = 5

Answer: junior 8, senior 15, veteran 5

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 10 cm and arc length 16 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{16}{10} = 1.6$

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times 1.6 = 80$

Answer: (a) 1.6 rad (b) 80 cm^2

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-5\mathbf{i} - 4\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(2\mathbf{i} + 2\mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 4.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-5\mathbf{i} - 4\mathbf{j}) + 4(2\mathbf{i} + 2\mathbf{j})$
$= 3\mathbf{i} + 4\mathbf{j}$

Speed = $\sqrt{(3^2 + 4^2)}$

Answer: $\mathbf{v} = (3\mathbf{i} + 4\mathbf{j})$ m s^{-1}; speed 5 m s^{-1}

4 Simplify using double angle formulae (All boards)

Simplify $\frac{1 + \cos 6x}{2 \cos 3x}$,

giving your answer as a single trigonometric term.

$1 + \cos 6x = 2\cos^2 3x$

so the expression is $\frac{2\cos^2 3x}{2 \cos 3x} = \cos 3x$

Answer: cos 3x

5 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -12 and the coefficient of x^2 is 60.
Find the values of n and k.

$nk = -12$, so $k = \frac{-12}{n}$

$\frac{n(n-1)}{2}k^2 = 60$, so $\frac{n(n-1)}{2} \times \frac{144}{n^2} = 60$

$\frac{(n-1) \times 144}{2n} = 60$, so $144(n - 1) = 120n$

$n = 6$, and $k = -2$

Answer: $n = 6$, $k = -2$

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6 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 2, and N lies on OB with ON : NB = 2 : 1. Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{1}{3}$(b − a)

OM = a + AM = $\frac{2a}{3} + \frac{b}{3}$

ON = $\frac{2b}{3}$, so NM = OM − ON = $\frac{2a}{3} - \frac{b}{3}$

Answer: OM = $\frac{2a}{3} + \frac{b}{3}$, NM = $\frac{2a}{3} - \frac{b}{3}$

7 Binomial one-tailed test (All boards)

A player says she scores with probability 0.5. In 25 shots she scores 10 times. Test, at the 1% level, whether the true proportion is lower than claimed.

H_0: p = 0.5, H_1: p < 0.5. Let X ~ B(25, 0.5) under H_0

P(X ≤ 10) = 0.2122

0.2122 > 0.01, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

8 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 18 kg is hinged at A and held horizontal by a light string attached at B. The string makes an angle θ with the rod, where sin θ = $\frac{4}{5}$. Take g = 9.8 m s^{-2}. Find the tension in the string.

Moments about A: T sin θ × 6 = mg × 3

T × $\frac{4}{5}$ × 6 = 176.4 × 3

T = 529.2 ÷ 4.8

Answer: 110.25 N

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{3}$ is irrational.

Assume $\sqrt{3} = \frac{a}{b}$, where a and b are integers with no common factor.

Then $a^2 = 3b^2$, so a^2 is a multiple of 3, so a is a multiple of 3.

Let a = 3k. Then $3^2k^2 = 3b^2$, so $b^2 = 3k^2$.

So b^2, and hence b, is a multiple of 3.

Then a and b have a common factor 3, a contradiction, so $\sqrt{3}$ is irrational.

Answer: $\sqrt{3}$ is irrational

10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation y = 2x − 2. The point P is (−7, −1).
(a) Find the line through P perpendicular to l, as ax + by + c = 0 with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 2, so the perpendicular has gradient $-\frac{1}{2}$

y + 1 = $-\frac{1}{2}$(x + 7), giving x + 2y + 9 = 0

(b) Solve y = 2x − 2 with x + 2y + 9 = 0: F = (−1, −4)

$PF^2 = (-6)^2 + 3^2 = 45$

Answer: (a) x + 2y + 9 = 0 (b) F(−1, −4), PF = $3\sqrt{5}$