Day 152 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of $(2 - x)^9$.

Term in $x^2 = 36 \times 2^7 \times (-x)^2$

$= 36 \times 128 \times 1\ x^2$

Answer: 4608

2 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 40 − 30 = 10. Class B: 50 − 30 = 20
(b) Medians: 35 (Class A) and 40 (Class B), so Class B has the higher median
IQRs: Class B has the larger IQR, so its scores are more spread out
Answer: (a) 10 and 20 (b) Class B has the higher median; Class B is more spread out (larger IQR), Class A more consistent

3 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^2 xe^{-2x}\,dx$.

Let u = x and $\frac{dv}{dx} = e^{-2x}$, so $\frac{du}{dx} = 1$ and $v = -\frac{e^{-2x}}{2}$

Integral = $[uv] - \int v\frac{du}{dx}\,dx$

$= \left[-\frac{xe^{-2x}}{2}\right]_0^2 - \int_0^2 -\frac{e^{-2x}}{2}\,dx$

$= \frac{e^4 - 5}{4e^4}$

Answer: $\mathbf{\frac{e^4 - 5}{4e^4}}$

4 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 60 kW. The resistance to motion is constant at 600 N.
Find the acceleration when the speed is 20 m s^{-1}.

P = Fv: F = 60000 ÷ 20 = 3000 N
F − R = ma: 3000 − 600 = 1000 a
Answer: 2.4 m s^{-2}

5 Simplify using double angle formulae (All boards)

Simplify $\frac{1 + \cos 2x}{2 \cos x}$,
giving your answer as a single trigonometric term.

$1 + \cos 2x = 2\cos^2 x$

so the expression is $\frac{2\cos^2 x}{2\cos x} = \cos x$

Answer: cos x

6 Two independent binomials (All boards)

X ~ B(11, 0.5) and Y ~ B(8, 0.25) are independent.
Find P(X = 7 and Y ≤ 3), to 3 significant figures.

P(X = 7) = 0.1611
P(Y ≤ 3) = 0.8862
Independent, so multiply: 0.1611 × 0.8862 = 0.143
Answer: 0.143

7 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(9 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 9: $(9 - 2x)^{\frac{1}{2}} = 9^{\frac{1}{2}}(1 + -\frac{2}{9}x)^{\frac{1}{2}} = 3(1 + -\frac{2}{9}x)^{\frac{1}{2}}$

Expand $(1 + -\frac{2}{9}x)^{\frac{1}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 3: coefficients of 1, x, x^2, x^3 are 3, $-\frac{1}{3}$, $-\frac{1}{54}$, $-\frac{1}{486}$

Valid when $|\frac{2}{9}x| < 1$, that is $|x| < \frac{9}{2}$

Answer: (a) $3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3 + ...$ (b) $|x| < \frac{9}{2}$

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x - \sec x - 1 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x - \sec x - 3 = 0$

$(2\sec x - 3)(\sec x + 1) = 0$, so $\sec x = \frac{3}{2}$ or -1

$\sec x = \frac{3}{2}$, so $\cos x = \frac{2}{3}$: $x = 48.2°, 311.8°$

$\sec x = -1$, so $\cos x = -1$: $x = 180.0°$

Answer: x = 48.2°, 180.0°, 311.8°

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{3}$ is irrational, prove by contradiction that $5 + 2\sqrt{3}$ is irrational.

Assume $5 + 2\sqrt{3} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $2\sqrt{3} = \frac{r}{s} - 5 = \frac{r - 5s}{s}$, so $\sqrt{3} = \frac{r - 5s}{2s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{3}$ being irrational, so $5 + 2\sqrt{3}$ is irrational.

Answer: $5 + 2\sqrt{3}$ is irrational

10 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is held on a rough plane ($\tan\alpha = \frac{3}{4}$, $\mu = 0.2$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 49 \times 4/5 = 39.2$
Friction (limiting) $F = \mu R = 7.84$
Moving up, friction acts down: $P = mg\sin\alpha + F$
$mg\sin\alpha = 29.4$
Answer: 37.24 N