Day 151 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 33 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 33$, $\Sigma fx = 146$, $\Sigma fx^2 = 698$

Mean = $\frac{146}{33} = 4.42$

Variance = $\frac{698}{33} - (4.4242)^2 = 1.578$

Standard deviation = 1.26

Answer: mean 4.42, standard deviation 1.26

2 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 10 s, (b) the distance between the stations.

(a) a = gradient = 12 ÷ 10
(b) Distance = area of trapezium = ½ × (54 + 40) × 12

Answer: (a) 1.2 m s^{-2} (b) 564 m

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "If a > b then $\frac{1}{a} < \frac{1}{b}$."

Find values of a and b to show this is false.

The result fails when a and b have opposite signs.

Take a = 3, b = −2: a > b, but $\frac{1}{a} = \frac{1}{3} > 0 > -\frac{1}{2} = \frac{1}{b}$.

Answer: a = 3, b = −2

4 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 4x - 9$.
Show that f(x) = 0 has a root between x = 2.5 and x = 3.

f(2.5) = −3.38
f(3) = 6.00
f is continuous and changes sign, so there is a root in the interval

Answer: f(2.5) < 0 and f(3) > 0: sign change

5 Poisson distribution (CIE)

X ~ Po(6), modelling the number of calls to a helpline per minute.
Find, to 3 significant figures,
(a) P(X ≥ 7)
(b) P(X = 7)

(a) P(X ≥ 7) = 1 − P(X ≤ 6) = 0.394

(b) P(X = 7) = $e^{-6} \times \frac{6^7}{7!} = 0.138$

Answer: (a) 0.394 (b) 0.138

6 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 10 m and mass 50 kg rests horizontally on supports at C and D, where AC = 1 m and BD = 1 m.

A particle of mass 20 kg is placed on the beam 2 m from B. Take g = 9.8 m s^{-2}.
Find the reactions at C and D.

Moments about C: $R_D \times 8 = 50\text{ g} \times 4 + 20\text{ g} \times 7$

$R_D = 3332 \div 8$

Resolve vertically: $R_C + R_D = 70\text{ g}$

Answer: R_C = 269.5 N; R_D = 416.5 N

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7 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cos\theta = \frac{20}{29}$ and $0 < \theta < 90°$,

find the exact value of $\sin 2\theta$.

$\sin\theta = \sqrt{1 - (\frac{20}{29})^2} = \frac{21}{29}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{21}{29} \times \frac{20}{29} = \frac{840}{841}$

Answer: $\mathbf{\frac{840}{841}}$

8 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t - 3$, $y = t^2 - 3t - 2$.
The line $y = 2x + 10$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 - 3t - 2 = -2t + 4$
$t^2 - t - 6 = 0$, so $(t - 3)(t + 2) = 0$
$t = -2$ or $t = 3$, giving the points $(-1, 8)$ and $(-6, -2)$
Answer: $(-1, 8)$ and $(-6, -2)$

9 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations

to find the value of $\lim_{\theta \to 0} \frac{\tan 2\theta \sin 3\theta}{1 - \cos 4\theta}$.

$\tan 2\theta \sin 3\theta \approx 2\theta \times 3\theta = 6\theta^2$

$1 - \cos 4\theta \approx \frac{16\theta^2}{2}$

Ratio $= \frac{6\theta^2}{\frac{16\theta^2}{2}} = \frac{3}{4}$

Answer: $\mathbf{\frac{3}{4}}$

10 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -4 and the coefficient of x^2 is 6.
Find the values of n and k.

$nk = -4$, so $k = \frac{-4}{n}$

$\frac{n(n-1)}{2}k^2 = 6$, so $\frac{n(n-1)}{2} \times \frac{16}{n^2} = 6$

$\frac{(n-1) \times 16}{2n} = 6$, so $16(n - 1) = 12n$

$n = 4$, and $k = -1$

Answer: $n = 4$, $k = -1$