Day 150 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 5 \geq 4x + 9$.

$-3x \geq 14$

Divide by -3 (reversing the inequality): $x \leq -\frac{14}{3}$

Answer: $x \leq -\frac{14}{3}$

2 Product rule (All boards)

Given that $y = x^2e^{-2x}$,

find $\frac{dy}{dx}$, and its value when $x = 1$.

Product rule with $u = x^2$, $v = e^{-2x}$

$u' = 2x$, $v' = -2e^{-2x}$

$\frac{dy}{dx} = u'v + uv' = -2x(x - 1)e^{-2x}$

When $x = 1$: $\frac{dy}{dx} = 0$

Answer: $\frac{dy}{dx} = -2x(x - 1)e^{-2x}$; value 0

3 Two independent binomials (All boards)

$X \sim B(10, 0.4)$ and $Y \sim B(9, 0.25)$ are independent.

Find $P(X = 3 \text{ and } Y \leq 2)$, to 3 significant figures.

$P(X = 3) = 0.2150$

$P(Y \leq 2) = 0.6007$

Independent, so multiply: $0.2150 \times 0.6007 = 0.129$

Answer: 0.129

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 4x + 2$ for $x \leq -1$.

Find the range of f.

$f(x) = (x - 2)^2 - 2$, with minimum at $x = 2$

$x = 2$ is not in the domain; f is decreasing on the domain, so the least value is $f(-1) = 7$.

Answer: $f(x) \geq 7$

5 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 15 kg rests on a rough horizontal floor, with coefficient of friction 0.5. A horizontal force of 59 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 15 \times 9.8 = 147$, maximum friction $\mu R = 0.5 \times 147 = 73.5$

$59 < 73.5$ so the box stays at rest

Friction balances the applied force: $F = 59$

Answer: Does not move; friction = 59 N

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, a quadratic.

(a) Find the coordinates of the local maximum point of $y = |f(x)|$.

(b) How many solutions has $|f(x)| = 14$?

(a) $y = |f(x)|$ reflects the part of the curve below the x-axis in the x-axis:

$(0, -9)$ becomes $(0, 9)$

(b) The maximum of the middle part is 9. Compare $k = 14$ with 9:

$k > 9$ gives 2 intersections with $y = k$.

Answer: (a) (0, 9) (b) 2

7 Optimisation: open box and tank (All boards)

An open tank has a square base of side x m and volume 4 m^3.

Show that its surface area is $S = x^2 + \frac{16}{x}$, and find the minimum value of S.

Height $h = \frac{4}{x^2}$, so $S = x^2 + 4xh = x^2 + \frac{16}{x}$

$\frac{dS}{dx} = 2x - \frac{16}{x^2} = 0$, so $x^3 = 8$ and $x = 2$

$\frac{d^2S}{dx^2} = 2 + \frac{32}{x^3} > 0$, so minimum

$S = 4 + 8 = 12\ m^2$

Answer: $S = 12\ m^2$ (when $x = 2$)

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 60 g; 50 packets have mean 61.3 g.
Test at the 5% level whether the true mean differs from 60.

$H_0: \mu = 60$, $H_1: \mu \neq 60$. Under H_0, $\overline{X} \sim N(60, \frac{5^2}{50})$

$z = \frac{61.3 - 60}{5/\sqrt{50}} = 1.838$

$P(Z > 1.838) = 0.0330 > 0.025$

Do not reject H_0: there is insufficient evidence that the mean has changed

Answer: do not reject H_0: insufficient evidence of a change in the mean

9 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

The function f is defined by $f(x) = \arctan x$.
State (a) the domain, (b) the range of f, in radians.

$\arctan x$ is the inverse of $\tan x$ restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$

The domain and range swap: (a) $x \in \mathbb{R}$ (all real x), (b) $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$

Answer: (a) $x \in \mathbb{R}$ (all real x) (b) $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$

10 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 24 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle θ with the rod, where $\sin\theta = \frac{4}{5}$. Take $g = 9.8\ m\ s^{-2}$. Find the tension in the string.

Moments about A: $T \sin\theta \times 4 = mg \times 2$

$T \times \frac{4}{5} \times 4 = 235.2 \times 2$

$T = 470.4 \div 3.2$

Answer: 147 N