Day 15 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:

(a) $y = f(\frac{x}{2})$ (b) y = 2f(x)

(a) $y = f(\frac{x}{2})$: A(−6, −2) maps to (−12, −2), B(−4, 3) maps to (−8, 3)

(b) y = 2f(x): A(−6, −2) maps to (−6, −4), B(−4, 3) maps to (−4, 6)

Answer: (a) (−12, −2), (−8, 3) (b) (−6, −4), (−4, 6)

2 Binomial one-tailed test (All boards)

A seed supplier says that 0.4 of seeds germinate. In a sample of 25 seeds, 13 germinate.
Test, at the 10% level, whether the true proportion is higher than claimed.

H_0: p = 0.4, H_1: p > 0.4. Let X ~ B(25, 0.4) under H_0

$P(X \geq 13) = 1 - P(X \leq 12) = 0.1538$

0.1538 > 0.1, so do not reject H_0

There is insufficient evidence that the proportion is higher than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is higher

3 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 16 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{8}{17}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 16$, so $P \times \frac{8}{17} = 16$

P = 34

Resolve vertically: $Q = P\sin\theta = 34 \times \frac{15}{17}$

Answer: P = 34 N; Q = 30 N

4 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 2.
Find the position vector of P.

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$

$\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{4} \\ \mathbf{-2} \\ \mathbf{0} \end{pmatrix}$

5 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{5}{x+4} + 2$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = -4$, and $y \to 2$ as $x \to \pm\infty$

Asymptotes: $x = -4$ and $y = 2$

(b) $x = 0$: $y = \frac{5}{4} + 2 = \frac{13}{4}$

$y = 0$: $\frac{5}{x+4} = -2$, so $x + 4 = -\frac{5}{2}$, $x = -\frac{13}{2}$

Answer: (a) $x = -4$, $y = 2$ (b) $(0, \frac{13}{4})$ and $(-\frac{13}{2}, 0)$

6 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 3x - 4$. Use the Newton-Raphson method with $x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{0}{6.000} = 1.0000$

$x_2 = 1.0000 - \frac{0}{6.000} = 1.0000$

Answer: $x_1 = 1.0000$, $x_2 = 1.0000$

7 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−13, 3) and B(−5, −9), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$169 + (h - 3)^2 = 25 + (h + 9)^2$

Expanding and cancelling h^2: $-6h + 178 = 18h + 106$

$h = 3$

$r^2 = CA^2 = 169$

Answer: centre (0, 3), $x^2 + (y - 3)^2 = 169$

8 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 8 kg lies on a rough horizontal table (coefficient of friction 0.5). It is joined by a light string over a smooth pulley to a particle of mass 6 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.5 \times 8g = 8a$

Hanging: $6g - T = 6a$

Add: $6g - 4g = 14a$

$a = 1.4$

$T = 6(9.8 - 1.4)$

Answer: $a = 1.4$ m s^{-2}; $T = 50.4$ N

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-3}{(x+2)(x+5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-3}{(x+2)(x+5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-3}{(x+2)(x+5)} \equiv \frac{A}{x+2} + \frac{B}{x+5}$, so $-3 \equiv A(x+5) + B(x+2)$
$x = -2$: $A = -1$; $x = -5$: $B = 1$

(b) $\int_0^4 (\frac{-1}{x+2} + \frac{1}{x+5})\,dx = [-\ln(x+2) + \ln(x+5)]_0^4$
$= -\ln\frac{6}{2} + \ln\frac{9}{5} = \ln(\frac{3}{5})$

Answer: (a) $A = -1$, $B = 1$ (b) $\ln(\frac{3}{5})$

10 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.25. It is tossed 30 times and lands heads 14 times. Test, at the 5% significance level, whether the probability of heads is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$. Under H_0, $X \sim B(30, 0.25)$
$P(X \geq 14) = 1 - P(X \leq 13) = 0.0082$
Two-tailed: compare with 0.025. $0.0082 < 0.025$, so reject H_0
There is sufficient evidence that the probability of heads is different from 0.25
Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.25