Day 149 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Work-energy: sliding to rest (CIE)

A block of mass 8 kg is projected along a rough horizontal surface with speed 7 m s^{-1} and comes to rest after sliding 10 m.

Take g = 9.8 m s^{-2}.

Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 8 \times 9.8 \times 10 = \frac{1}{2} \times 8 \times 7^2$

$784\,\mu = 196$

Answer: 0.25

2 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "8n + 5 is prime for every positive integer n."

Show that this statement is false by finding a counter-example.

Try n = 1, 2, 3, ... until the value is not prime.

n = 2 gives 8 × 2 + 5 = 21

21 = 3 × 7, which is not prime.

Answer: n = 2 (value 21 is not prime)

3 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 10 \\ 1 \end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -4 \\ 8 \\ 1 \end{pmatrix}$

$|\text{AB}| = \sqrt{4^2 + 8^2 + 1^2} = \sqrt{81} = 9$

Answer: AB = $\begin{pmatrix} -4 \\ 8 \\ 1 \end{pmatrix}$, distance 9

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

12 pairs of height and arm span give r = 0.559.

One-tailed critical value (1%): 0.6581.

Test for positive correlation.

H_0: $\rho = 0$, H_1: $\rho > 0$

r = 0.559, critical value 0.6581; r is not beyond the critical value in the right direction

r is not in the critical region, so do not reject H_0

Conclusion: there is insufficient evidence of positive correlation between height and arm span

Answer: do not reject H_0: there is insufficient evidence of positive correlation between height and arm span

5 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg lies on a rough horizontal table (coefficient of friction 0.4). It is joined by a light string over a smooth pulley to a particle of mass 4 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.4 \times 3g = 3a$

Hanging: $4g - T = 4a$

Add: $4g - 1.2g = 7a$

$a = 3.92$

$T = 4(9.8 - 3.92)$

Answer: a = 3.92 m s^{-2}; T = 23.52 N

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6 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + 3x + y^2 + y - 2xy = 30$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-3, 3)$.

Differentiate with respect to x: $2x + 2y\frac{dy}{dx} + \frac{dy}{dx} + 3 - 2x\frac{dy}{dx} - 2y = 0$

$\frac{dy}{dx} = \frac{2y - 2x - 3}{2y + 1 - 2x}$

At $(-3, 3)$: gradient $= \frac{9}{13}$

Answer: $\frac{dy}{dx} = \frac{2y - 2x - 3}{2y + 1 - 2x}$; gradient $\frac{9}{13}$

7 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(1, −3) and B(4, 12), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$1 + (h + 3)^2 = 16 + (h - 12)^2$

Expanding and cancelling h^2: $6h + 10 = -24h + 160$

$h = 5$

$r^2 = CA^2 = 65$

Answer: centre (0, 5), $x^2 + (y - 5)^2 = 65$

8 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $12\sin^2 x - 5\sin x - 2 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(3\sin x - 2)(4\sin x + 1) = 0$, so $\sin x = \frac{2}{3}$ or $\sin x = -\frac{1}{4}$

$\sin x = \frac{2}{3}$: $x = 41.8°, 138.2°$

$\sin x = -\frac{1}{4}$: $x = 194.5°, 345.5°$

Answer: x = 41.8°, 138.2°, 194.5°, 345.5°

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 48) = 0.1587 and P(X > 72) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$

$48 = \mu + (-0.9998)\sigma$ and $72 = \mu + 0.9998\sigma$

Subtract: $24 = 2.000\,\sigma$, so $\sigma = 12.0$

$\mu = 48 - (-0.9998)(12.00) = 60.0$

Answer: $\mu = 60.0$, $\sigma = 12.0$

10 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(8 + 4x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 8: $(8 + 4x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1 + \frac{1}{2}x)^{\frac{1}{3}} = 2(1 + \frac{1}{2}x)^{\frac{1}{3}}$

Expand $(1 + \frac{1}{2}x)^{\frac{1}{3}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 2: coefficients of 1, x, x^2, x^3 are 2, $\frac{1}{3}$, $-\frac{1}{18}$, $\frac{5}{324}$

Valid when $|\frac{1}{2}x| < 1$, that is $|x| < 2$

Answer: (a) $2 + \frac{1}{3}x - \frac{1}{18}x^2 + \frac{5}{324}x^3 + ...$ (b) $|x| < 2$