Day 148 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 20 kg rests on a rough horizontal floor, with coefficient of friction 0.4. A horizontal force of 63 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 20 \times 9.8 = 196$, maximum friction $\mu R = 0.4 \times 196 = 78.4$

$63 < 78.4$ so the box stays at rest

Friction balances the applied force: $F = 63$

Answer: Does not move; friction = 63 N

2 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Values of $y = f(x)$ are shown.

x: 2, 2.5, 3, 3.5, 4

y: 1.9, 0.8, 0.9, 1.7, 2.2

Estimate $\int_2^4 f(x)\,dx$ using the trapezium rule.

$h = 0.5$

$\text{Area} \approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2}[1.9 + 2.2 + 2(0.8 + 0.9 + 1.7)]$

$= 2.725$

Answer: 2.725

3 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(2 - 2x)^7$ in ascending powers of x, up to and including the term in x^3.

$(2 - 2x)^7 = 2^7 + 7 \times 2^6(-2x) + 21 \times 2^5(-2x)^2 + 35 \times 2^4(-2x)^3$

$= 128 - 896x + 2688x^2 - 4480x^3 + \ldots$

Answer: $128 - 896x + 2688x^2 - 4480x^3 + \ldots$

4 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 2 kg and 12 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.

Take $g = 9.8$ m s^{-2}.

Find the acceleration and the tension in the string.

Heavier: $12g - T = 12a$

Lighter: $T - 2g = 2a$

Add: $10g = 14a$, so $a = 7$

$T = 2(9.8 + 7)$

Answer: $a = 7$ m s^{-2}; $T = 33.6$ N

5 Binomial one-tailed test (All boards)

A player says she scores with probability 0.25. In 30 shots she scores 6 times.

Test, at the 5% level, whether the true proportion is lower than claimed.

H_0: $p = 0.25$, H_1: $p < 0.25$. Let $X \sim B(30, 0.25)$ under H_0

$P(X \leq 6) = 0.3481$

$0.3481 > 0.05$, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $5x + 4y = -25$ and line l_2 has equation $5x - 8y = 35$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(-1, -5)$
l_1 meets $y = 0$ at $x = -5$, and l_2 meets $y = 0$ at $x = 7$
Base = 12, height = 5
Area = $\frac{1}{2} \times$ base $\times$ height = 30

Answer: 30

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-3}{(x+3)(x+4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-3}{(x+3)(x+4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-3}{(x+3)(x+4)} \equiv \frac{A}{x+3} + \frac{B}{x+4}$, so $-3 \equiv A(x+4) + B(x+3)$
$x = -3$: $A = -3$; $x = -4$: $B = 3$
(b) $\int_0^4 \left(\frac{-3}{x+3} + \frac{3}{x+4}\right) dx = [-3\ln(x+3) + 3\ln(x+4)]_0^4$
$= -3\ln\frac{7}{3} + 3\ln\frac{8}{4} = \ln\left(\frac{216}{343}\right)$

Answer: (a) $A = -3$, $B = 3$ (b) $\ln\left(\frac{216}{343}\right)$

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 6 ml. Set mean 50 ml; 40 bottles have mean 51.3 ml.
Test at the 5% level whether the true mean is greater than 50.

H_0: $\mu = 50$, H_1: $\mu > 50$. Under H_0, $\overline{X} \sim N\left(50, \frac{6^2}{40}\right)$
$z = \frac{51.3 - 50}{6/\sqrt{40}} = 1.370$
$P(Z > 1.370) = 0.0853 > 0.05$
Do not reject H_0: there is insufficient evidence that the mean has increased

Answer: do not reject H_0: insufficient evidence of an increase in the mean

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 2x + 18$ meets the circle $(x+3)^2 + (y+3)^2 = 50$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = 2x + 18$: $(x+3)^2 + (2x+21)^2 = 50$
$x^2 + 18x + 80 = 0$
$(x+8)(x+10) = 0$, so $x = -10$ or $x = -8$
$y = 2x + 18$: A $= (-10, -2)$, B $= (-8, 2)$
$AB^2 = 2^2 + 4^2 = 20$

Answer: A$(-10, -2)$, B$(-8, 2)$, AB $= 2\sqrt{5}$

10 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = xe^{-x}$, and determine its nature.

$\frac{dy}{dx} = (1 - x)e^{-x} = 0$

$x = 1$

$\frac{d^2y}{dx^2} = (x - 2)e^{-x}$

At $x = 1$: $\frac{d^2y}{dx^2} = -\frac{1}{e} < 0$, so a maximum

Answer: $(1, \frac{1}{e})$ maximum