Day 147 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 200 and common ratio 0.5.
Find the smallest n for which the nth term is less than 0.01.

$200 \times 0.5^{n-1} < 0.01$
$(n - 1) \lg 0.5 < \lg(0.01 \div 200)$
lg 0.5 is negative, so dividing reverses the inequality:
$n - 1 > \lg(0.01 \div 200) \div \lg 0.5 = 14.288$, so $n > 15.288$
The smallest integer n is 16
Answer: n = 16

2 Chain rule (All boards)

Given that $y = (3x^2 + 4)^3$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 0.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = 18x(3x^2 + 4)^2$
When x = 0: gradient = 0
Answer: $\frac{dy}{dx} = 18x(3x^2 + 4)^2$; gradient 0

3 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 7 m s^{-1} to 16 m s^{-1} while covering 103.5 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $256 = 49 + 207\,a$, so $a = 1$
(b) $v = u + at$: $16 = 7 + 1\,t$
$t = 9 \div 1 = 9$
Answer: (a) 1 m s^{-2} (b) 9 s

4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 5 consecutive integers is always a multiple of 5.

Let the integers be n, n + 1, n + 2, n + 3, n + 4
Sum = 5n + 10
= 5(n + 2)
n is an integer, so (n + 2) is an integer, so the sum is a multiple of 5.
Answer: sum = 5(n + 2), a multiple of 5

5 Laws of logarithms: solving equations (All boards)

Solve $\log_5(x + 23) - \log_5(x - 1) = 1$.

$\log_5(\frac{x + 23}{x - 1}) = 1$, so $\frac{x + 23}{x - 1} = 5^1 = 5$
$x + 23 = 5(x - 1) = 5x - 5$
$4x = 28$, so $x = 7$
Answer: x = 7

6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 + 4x + 5$.
C is reflected in the x-axis.
Find its new equation.

Reflection in the x-axis: replace y by −y.
When reflected in the x-axis: $y = -x^2 - 4x - 5$
Answer: $y = -x^2 - 4x - 5$

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7 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 30t - 5t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 30 - 10t$
Maximum speed when $a = 0$: $t = 30 \div 10 = 3$
$v = 30 \times 3 - 5 \times 3^2$
Answer: 45 m s^{-1} at t = 3 s

8 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.3. In a sample of 30 students, 6 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(30, 0.3)$
$P(X \leq 6) = 0.1595$
Two-tailed: compare with 0.025. $0.1595 > 0.025$, so do not reject H_0
There is insufficient evidence that the probability is different from 0.3
Answer: do not reject H_0: insufficient evidence that the probability is different from 0.3

9 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{4}{5}$ and $\sin B = \frac{7}{25}$, where A and B are acute,
find the exact value of $\sin(A + B)$.

$\cos A = \frac{3}{5}$ and $\cos B = \frac{24}{25}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$= \frac{4}{5} \times \frac{24}{25} + \frac{3}{5} \times \frac{7}{25} = \frac{117}{125}$
Answer: $\frac{117}{125}$

10 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with $P(A) = 0.3$ and $P(A \cup B) = 0.55$.
(a) Find P(B).
(b) Find $P(A' \cap B')$.

(a) Mutually exclusive: $P(A \cup B) = P(A) + P(B)$
$P(B) = 0.55 - 0.3 = 0.25$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.45$
Answer: (a) 0.25 (b) 0.45