Day 146 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 5 kg is moving on a smooth horizontal surface at 0 m s^{-1} when a constant horizontal force of 24 N, in the direction of motion, acts for 4 s.
Find the speed of the particle at the end of this time and the distance it moves.

F = ma: a = 24 ÷ 5 = 4.8
v = u + at = 0 + 4.8 × 4
$s = ut + \frac{1}{2}at^2$
Answer: 19.2 m s^{-1}; 38.4 m

2 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value, in radians, of
(a) $\arcsin(-\frac{1}{2})$ (b) $\arctan(-\frac{\sqrt{3}}{3})$

(a) arcsin lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$, so the value is $-\frac{\pi}{6}$

(b) arctan lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$, and $\tan(-\frac{\pi}{6}) = -\frac{\sqrt{3}}{3}$, so the value is $-\frac{\pi}{6}$

Answer: (a) $-\frac{\pi}{6}$ (b) $-\frac{\pi}{6}$

3 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x+4} = 8^{4x-2}$.

Write both sides as powers of 2: $2^{2x+4} = 2^{3(4x-2)}$
Equate the indices: 2x + 4 = 12x − 6
−10x = −10
x = 1
Answer: x = 1

4 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2 + 3t)\mathbf{i} + (3t^3)\mathbf{j}$ metres.
Find the velocity when t = 2, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t + 3)\mathbf{i} + (9t^2)\mathbf{j}$
t = 2: $\mathbf{v} = (15)\mathbf{i} + (36)\mathbf{j}$
Speed = $\sqrt{(15^2 + 36^2)}$
Answer: $\mathbf{v} = (15)\mathbf{i} + (36)\mathbf{j}$ m s^{-1}; speed 39 m s^{-1}

5 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 48, is a bar 2 cm wide, 12 cm tall.
(a) Find the bar height for width 30, frequency 24.
(b) Find the frequency of a 1 cm by 2.5 cm bar.

Area of bar ∝ frequency: 2 × 12 = 24 cm^2 represents 48
so 1 unit of frequency = 0.5 cm^2
(a) Width = 3 cm, area = 0.5 × 24 = 12 cm^2, height = 12 ÷ 3 = 4 cm
(b) Area = 1 × 2.5 = 2.5 cm^2, frequency = 2.5 ÷ 0.5 = 5
Answer: (a) 4 cm (b) 5

6 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form 7k + 4, where k is an integer.

Write the integer as n = 7q + j, where j = 0, 1, ..., 6.
n^3 leaves the same remainder as j^3 when divided by 7.
j = 0, 1, 2, 3, 4, 5, 6 give remainders 0, 1, 1, 6, 1, 6, 6.
The only possible remainders are 0, 1, 6, never 4.
Answer: possible remainders 0, 1, 6, never 4

7 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}7\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\0\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}3\\3\\0\end{pmatrix} + \mu\begin{pmatrix}-1\\2\\-1\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $7 + 3\lambda = 3 + (-1)\mu$

j: $1 + 0\lambda = 3 + 2\mu$

Solving i and j: $\lambda = -1$, $\mu = -1$

k check: $1 + 0(-1) = 1$ and $0 + (-1)(-1) = 1$, equal

Point: $\begin{pmatrix}4\\1\\1\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{4}\\\mathbf{1}\\\mathbf{1}\end{pmatrix}$

8 Union, conditional probability and independence (All boards)

P(A) = 0.4, P(B) = 0.6, P(A ∩ B) = 0.05.

Find (a) P(A ∪ B), (b) P(A | B).

(c) Are A and B independent?

(a) $P(A \cup B) = 0.4 + 0.6 - 0.05 = 0.95$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.6} = \frac{1}{12}$

(c) $P(A)P(B) = 0.4 \times 0.6 = 0.24$, which does not equal $P(A \cap B)$

Answer: (a) 0.95 (b) $\frac{1}{12}$ (c) not independent

9 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cosec 60°,

giving your answer with a rational denominator.

$\text{cosec } 60° = \frac{1}{\sin 60°}$

$\sin 60° = \frac{\sqrt{3}}{2}$, so $\text{cosec } 60° = \frac{1}{\frac{\sqrt{3}}{2}}$

Rationalising the denominator gives $\frac{2\sqrt{3}}{3}$

Answer: $\mathbf{\frac{2\sqrt{3}}{3}}$

10 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(5 - 4x)^{-1}$ in ascending powers of x up to and including the term in x^3.

(b) State the range of values of x for which the expansion is valid.

Take out 5: $(5 - 4x)^{-1} = 5^{-1}(1 + -\frac{4}{5}x)^{-1} = \frac{1}{5}(1 + -\frac{4}{5}x)^{-1}$

Expand $(1 + -\frac{4}{5}x)^{-1} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Multiplying by $\frac{1}{5}$: coefficients of 1, x, x^2, x^3 are $\frac{1}{5}$, $\frac{4}{25}$, $\frac{16}{125}$, $\frac{64}{625}$

Valid when $|\frac{4}{5}x| < 1$, that is $|x| < \frac{5}{4}$

Answer: (a) $\frac{1}{5} + \frac{4}{25}x + \frac{16}{125}x^2 + \frac{64}{625}x^3 + \ldots$ (b) $|x| < \frac{5}{4}$