Day 145 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Laws of logarithms: solving equations (All boards)

Solve $\log_5 x + \log_5(x + 20) = 3$.

$\log_5(x(x + 20)) = 3$, so $x(x + 20) = 5^3 = 125$

$x^2 + 20x - 125 = 0$, so $(x - 5)(x + 25) = 0$

$x > 0$ for the logs to exist, so $x = 5$

Answer: $x = 5$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-5 \leq 5x - 2 < 3$.

Subtract -2 from each part: $-3 \leq 5x < 5$

Divide each part by 5: $-\frac{3}{5} \leq x < 1$

Answer: $-\frac{3}{5} \leq x < 1$

3 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, goals scored by a player in 12 seasons:
13, 14, 21, 22, 24, 28, 31, 33, 34, 34, 38, 39.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = \frac{43}{2}$, $Q_3 = 34$, IQR $= \frac{25}{2}$

Lower limit $= \frac{43}{2} - 1.5 \times \frac{25}{2} = \frac{11}{4}$

Upper limit $= 34 + 1.5 \times \frac{25}{2} = \frac{211}{4}$

No value lies outside these limits.

Answer: $Q_1 = \frac{43}{2}$, $Q_3 = 34$, IQR $= \frac{25}{2}$; no outliers

4 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 6 s, (b) the distance between the stations.

(a) a = gradient = 15 ÷ 6

(b) Distance = area of trapezium = ½ × (20 + 10) × 15

Answer: (a) 2.5 m s^{-2} (b) 225 m

5 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve $y = f(x)$, where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at $x = -1$, 1 and 2, so $y = k(x + 1)(x - 1)(x - 2)$

The curve passes through (0, 2): $k \times 1 \times (-1) \times (-2) = 2$

So $k = 1$

Answer: $f(x) = (x + 1)(x - 1)(x - 2)$

6 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 7 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 7 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 7g = 7a$

Hanging: $7g - T = 7a$

Add: $7g - 1.4g = 14a$

$a = 3.92$

$T = 7(9.8 - 3.92)$

Answer: $a = 3.92$ m s^{-2}; $T = 41.16$ N

Day 145 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{9}{41}$ and $\sin B = \frac{4}{5}$, where A is obtuse and B is acute,

find the exact value of $\cos(A - B)$.

$\cos A = -\frac{40}{41}$ and $\cos B = \frac{3}{5}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (-\frac{40}{41}) \times \frac{3}{5} + \frac{9}{41} \times \frac{4}{5} = -\frac{84}{205}$

Answer: $-\frac{84}{205}$

8 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.3. In a sample of 20 students, 1 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(20, 0.3)$

$P(X \leq 1) = 0.0076$

Two-tailed: compare with 0.025. $0.0076 < 0.025$, so reject H_0

There is sufficient evidence that the probability is different from 0.3

Answer: reject H_0: sufficient evidence that the probability is different from 0.3

9 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (3e^{2x} + \frac{3}{x})\, dx$.

$\int (3e^{2x} + \frac{3}{x})\, dx = \frac{3e^{2x}}{2} + 3 \ln x$

$= [\frac{3e^{2x}}{2} + 3 \ln x]_1^2$

$= \ln 8 + \frac{3e^4}{2} - \frac{3e^2}{2}$

Answer: $\ln 8 + \frac{3e^4}{2} - \frac{3e^2}{2}$

10 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 4| = |3x - 6|$.

Either $x - 4 = 3x - 6$ or $x - 4 = -(3x - 6)$

$x - 4 = 3x - 6$ gives $x = 1$

$x - 4 = -(3x - 6)$ gives $x = \frac{5}{2}$

Answer: $x = 1$ and $x = \frac{5}{2}$