Day 144 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A survey covers 28 urban, 60 suburban and 64 rural households.
A stratified sample of size 38 is taken. Find the number selected from each group.

Total = 28 + 60 + 64 = 152

Sampling fraction = 38/152 = $\frac{1}{4}$

urban: 28 ÷ 4 = 7
suburban: 60 ÷ 4 = 15
rural: 64 ÷ 4 = 16

Answer: urban 7, suburban 15, rural 16

2 Chain rule (All boards)

Given that y = sin 2x,

find $\frac{dy}{dx}$, and the gradient of the curve when x = $\frac{\pi}{4}$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx}$ = 2 cos 2x

When x = $\frac{\pi}{4}$: gradient = 0

Answer: $\frac{dy}{dx}$ = 2 cos 2x; gradient 0

3 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation y = $\frac{4}{x - 3} - 1$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = 3, and y → −1 as x → ±∞
Asymptotes: x = 3 and y = −1

(b) x = 0: y = $-\frac{4}{3} - 1 = -\frac{7}{3}$

y = 0: $\frac{4}{x - 3}$ = 1, so x − 3 = 4, x = 7

Answer: (a) x = 3, y = −1 (b) $(0, -\frac{7}{3})$ and (7, 0)

4 Total probability and Bayes (All boards)

R: made by machine A; L: item is faulty.
P(R) = 0.2, P(L | R) = 0.4, P(L | R') = 0.25.
(a) Find P(L).
(b) Find P(R | L).

(a) P(L) = 0.2 × 0.4 + 0.8 × 0.25 = 0.08 + 0.2 = 0.28

(b) P(R | L) = $\frac{0.2 \times 0.4}{0.28} = \frac{0.08}{0.28}$ = 0.286

Answer: (a) 0.28 (b) $\frac{2}{7}$

5 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

The curve touches the x-axis at x = 2 (repeated root) and crosses at x = −2
So y = k(x − 2)^2(x + 2)
The curve passes through (0, 8): k × 4 × 2 = 8
So k = 1

Answer: f(x) = (x − 2)^2(x + 2)

6 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 98 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 24 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 24 = 235.2
Speed = $\sqrt{(98^2 + 235.2^2)}$ = 254.8
tan α = 235.2 ÷ 98, α = 67.4°
Answer: 254.8 m s^{-1} at 67.4° below horizontal

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that cos θ + sin θ ≡ $\sqrt{2}$ cos(θ − 45.0°),
solve cos θ + sin θ = −0.5 for 0 ≤ θ ≤ 360°. (1 d.p.)

$\sqrt{2}$ cos(θ − 45.0°) = −0.5, so cos(θ − 45.0°) = $\frac{-0.5}{\sqrt{2}}$
θ − 45.0° = ±110.7°
θ = 45.0° + 110.7° = 155.7°
θ = 45.0° − 110.7° = −65.7°, so θ = 294.3°
Answer: θ = 155.7°, 294.3°

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 3x − 4y = k is a tangent to the circle
$(x - 1)^2 + (y + 4)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, −4).
A unit step in this direction is (3, −4)/5, so the points of contact are
(1, −4) ± 10 × (3, −4)/5 = (1, −4) ± (6, −8)
T = (7, −12) or (−5, 4), and k = 3x − 4y gives:
k = 69 or k = −31
Answer: k = 69 or k = −31

9 Quadratic equations in sin x, cos x, tan x (All boards)

Solve 3 $\cos^2 x$ + 2 cos x − 1 = 0 for 0 ≤ x ≤ 360°,
giving answers to 1 d.p. where necessary.

(cos x + 1)(3 cos x − 1) = 0, so cos x = −1 or cos x = $\frac{1}{3}$
cos x = −1: x = 180.0°
cos x = $\frac{1}{3}$: x = 70.5°, 289.5°
Answer: x = 70.5°, 180.0°, 289.5°

10 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 20 ÷ 4
(b) Distance = area of trapezium = ½ × (22 + 12) × 20
Answer: (a) 5 m s^{-2} (b) 340 m