Day 143 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 (x + 3)e^x \, dx$.

Let $u = x + 3$ and $\frac{dv}{dx} = e^x$, so $\frac{du}{dx} = 1$ and $v = e^x$

Integral = $[uv] - \int v\frac{du}{dx} \, dx$

$= [(x + 3)e^x]_0^1 - \int_0^1 e^x \, dx$

$= 3e - 2$

Answer: $3e - 2$

2 Probability without replacement (All boards)

A bag has 8 white and 9 black sweets; two are taken without replacement.
(a) Find P(same colour).
(b) Given they are the same colour, find P(both white).

(a) P(white white) = $\frac{8}{17} \times \frac{7}{16} = \frac{7}{34}$

P(black black) = $\frac{9}{17} \times \frac{8}{16} = \frac{9}{34}$

P(same) = $\frac{7}{34} + \frac{9}{34} = \frac{8}{17}$

(b) P = $\frac{7}{34} \div \frac{8}{17} = \frac{7}{16}$

Answer: (a) $\frac{8}{17}$ (b) $\frac{7}{16}$

3 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin\theta = \frac{5}{13}$ and $90° < \theta < 180°$,

find the exact value of $\tan 2\theta$.

$\cos\theta = \pm\sqrt{1 - (\frac{5}{13})^2} = -\frac{12}{13}$ (θ is obtuse, so $\cos\theta < 0$)

$\sin 2\theta = -\frac{120}{169}$ and $\cos 2\theta = \frac{119}{169}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{120}{119}$

Answer: $-\frac{120}{119}$

4 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 2 kg is acted on by forces $(5\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(5\mathbf{i} + 6\mathbf{j}) + \mathbf{P} = 2(-\mathbf{i} + 2\mathbf{j}) = (-2\mathbf{i} + 4\mathbf{j})$

$\mathbf{P} = -7\mathbf{i} - 2\mathbf{j}$

$|\mathbf{P}| = \sqrt{(-7^2 + -2^2)}$

Answer: $\mathbf{P} = (-7\mathbf{i} - 2\mathbf{j})$ N; $|\mathbf{P}| = 7.28$ N

5 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x+3}{(x+1)(x+2)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{2x+3}{(x+1)(x+2)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x+3}{(x+1)(x+2)} \equiv \frac{A}{x+1} + \frac{B}{x+2}$, so $2x+3 \equiv A(x+2) + B(x+1)$

$x = -1$: $A = 1$; $x = -2$: $B = 1$

(b) $\int_0^2 (\frac{1}{x+1} + \frac{1}{x+2})\,dx = [\ln(x+1) + \ln(x+2)]_0^2$

$= \ln\frac{3}{1} + \ln\frac{4}{2} = \ln(6)$

Answer: (a) A = 1, B = 1 (b) ln(6)

6 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 3 and Var(X) = 5.
Find (a) E(3X − 1),
(b) Var(3X − 1), (c) $E(X^2)$.

(a) E(aX + b) = aE(X) + b = 3 × 3 − 1 = 8

(b) $Var(aX + b) = a^2Var(X) = 9 \times 5 = 45$

(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 5 + 9 = 14$

Answer: (a) 8 (b) 45 (c) 14

7 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 4x - 1$ and $g(x) = x^2 - 5$ for $x \in \mathbb{R}$.
(a) Find fg(x).
(b) Solve gf(x) = 284.

$fg(x) = f(x^2 - 5) = 4(x^2 - 5) - 1 = 4x^2 - 21$

$gf(x) = (4x - 1)^2 - 5 = 284$

$(4x - 1)^2 = 289$, so $4x - 1 = \pm 17$

$x = -4$ or $x = \frac{9}{2}$

Answer: (a) $fg(x) = 4x^2 - 21$ (b) $x = -4$ or $x = \frac{9}{2}$

8 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + t^2$ metres.
Find (a) the average velocity over the first 6 seconds, (b) the velocity at t = 6.

(a) s(6) = 684, so average velocity = 684 ÷ 6

(b) $v = ds/dt = 9t^2 + 2t$, v(6) = 336

Answer: (a) 114 m s^{-1} (b) 336 m s^{-1}

9 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(7, −5) and B(8, −2), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$49 + (h + 5)^2 = 64 + (h + 2)^2$

Expanding and cancelling h^2: $10h + 74 = 4h + 68$

$h = -1$

$r^2 = CA^2 = 65$

Answer: centre (0, −1), $x^2 + (y + 1)^2 = 65$

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10 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $3^{2x} + 3\times3^{x} - 4 = 0$.

Let $y = 3^x$. Then $3^{2x} = y^2$, so $y^2 + 3y - 4 = 0$
$(y - 1)(y + 4) = 0$
$y = 1$ or $y = -4$
$3^x > 0$ for all x, so $y = -4$ is rejected. $3^x = 1$ gives $x = 0$.
Answer: x = 0