Day 142 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2x - 6 \geq x + 3$.

$-3x \geq 9$

Divide by -3 (reversing the inequality): $x \leq -3$

Answer: $x \leq -3$

2 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 6e^{3x}\,dx$,

(b) the exact value of $\int_0^1 6e^{3x}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $2e^{3x} + c$

(b) $[2e^{3x}]_0^1$

$= 2e^3 - 2$

Answer: (a) $2e^{3x} + c$ (b) $2e^3 - 2$

3 Binomial expected frequency (All boards)

Each of 8 people independently supports a team with probability 0.6; X is the number of supporters. This is done 500 times. Estimate the number of times that X = 6.

$X \sim B(8, 0.6)$

$P(X = 6) = {}^8C_6 \times (0.6)^6 \times (0.4)^2 = 0.2090$

Expected $= 500 \times 0.2090 = 104.5$

Answer: about 105 times

4 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $16^{-\frac{1}{4}}$.

$16 = 2^4$, so $16^{-\frac{1}{4}} = (2^4)^{-\frac{1}{4}} = 2^{-1}$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$

5 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{24}{25}$ and $\sin B = \frac{8}{17}$, where A is obtuse and B is acute, find the exact value of $\cos(A + B)$.

$\cos A = -\frac{7}{25}$ and $\cos B = \frac{15}{17}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= (-\frac{7}{25}) \times \frac{15}{17} - \frac{24}{25} \times \frac{8}{17} = -\frac{297}{425}$

Answer: $-\frac{297}{425}$

6 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point 98 m above horizontal ground. Take g = 9.8 m s^{-2}. Find the time before it hits the ground.

Taking up as positive: $-98 = 4.9\,t - 4.9t^2$

$4.9t^2 - 4.9\,t - 98 = 0$

$t = (4.9 \pm \sqrt{1944.81}) \div 9.8 = (4.9 \pm 44.1) \div 9.8$

Positive root only.

Answer: 5 s

7 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x + 7y = 3$ and line l_2 has equation $x + 4y = 0$.

The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(-4, 1)$

l_1 meets $y = 0$ at $x = 3$, and l_2 meets $y = 0$ at $x = 0$

Base = 3, height = 1

Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{3}{2}$

Answer: $\frac{3}{2}$

8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.5. It is tossed 30 times and lands heads 11 times.

Test, at the 5% significance level, whether the probability of heads is different from 0.5.

H_0: $p = 0.5$, H_1: $p \neq 0.5$. Under H_0, $X \sim B(30, 0.5)$

$P(X \leq 11) = 0.1002$

Two-tailed: compare with 0.025. $0.1002 > 0.025$, so do not reject H_0

There is insufficient evidence that the probability of heads is different from 0.5

Answer: do not reject H_0: insufficient evidence that the probability of heads is different from 0.5

9 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 2)\mathbf{i} + (5t - 20)\mathbf{j}$ m s^{-1}.

Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0

$5t - 20 = 0$, so $t = 4$

$\mathbf{v} = (3 \times 4^2 + 2)\mathbf{i} = 50\mathbf{i}$

Answer: t = 4 s, speed 50 m s^{-1}

10 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 48 cm and the sides are folded up to make an open box.

Find x for the greatest volume, and that volume.

$V = x(48 - 2x)^2 = 4x^3 - 192x^2 + 2304x$

$\frac{dV}{dx} = 12x^2 - 384x + 2304 = (6x - 48)(2x - 48) = 0$

$x < 24$, so $x = 8$

$\frac{d^2V}{dx^2} = 24x - 384 = -192 < 0$ at $x = 8$, so maximum

$V = 8 \times 32^2 = 8192$

Answer: x = 8, V = 8192 cm^3