Day 141 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -3x + 2$ and $g(x) = x^2 - 3$ for $x \in \mathbb{R}$.
(a) Find $fg(x)$.
(b) Solve $gf(x) = 1$.

$fg(x) = f(x^2 - 3) = -3(x^2 - 3) + 2 = -3x^2 + 11$
$gf(x) = (-3x + 2)^2 - 3 = 1$
$(-3x + 2)^2 = 4$, so $-3x + 2 = \pm 2$
$x = 0$ or $x = \frac{4}{3}$

Answer: (a) $fg(x) = -3x^2 + 11$ (b) $x = 0$ or $x = \frac{4}{3}$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 30 cm^2 and angle 0.6 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 30$, so $r^2 = \frac{2 \times 30}{0.6} = 100$, $r = 10$
Perimeter $= 2r + r\theta = 20 + 6 = 26$ cm
Answer: (a) 10 cm (b) 26 cm

3 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $2x^2 + 20x + 57 > 0$ for all real values of x.

Complete the square: $2x^2 + 20x + 57 = 2(x + 5)^2 + 7$
$(x + 5)^2 \geq 0$ for all real x, so $2(x + 5)^2 \geq 0$.
Adding 7: the expression is ≥ 7, so > 0 for all real x.
Answer: $2x^2 + 20x + 57 > 0$ for all x

4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-6\mathbf{i} - 2\mathbf{j})$ N and $(-2\mathbf{i} + 8\mathbf{j})$ N act on a particle of mass 4 kg.
Find the magnitude of the acceleration of the particle.

Resultant $= (-6\mathbf{i} - 2\mathbf{j}) + (-2\mathbf{i} + 8\mathbf{j}) = -8\mathbf{i} + 6\mathbf{j}$
$|R| = \sqrt{(-8^2 + 6^2)} = 10$ N
$a = F/m = 10 \div 4$
Answer: 2.5 m s^{-2}

5 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.5 of its sweets are red. A sample of 25 sweets has 10 red.
Test, at the 1% level, whether the true proportion is lower than claimed.

H_0: $p = 0.5$, H_1: $p < 0.5$. Let $X \sim B(25, 0.5)$ under H_0
$P(X \leq 10) = 0.2122$
$0.2122 > 0.01$, so do not reject H_0
There is insufficient evidence that the proportion is lower than claimed
Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the form $1 + p\theta + q\theta^2$ for
$\sin 3\theta + \cos 4\theta + 2\tan\theta$.

$\sin 3\theta \approx 3\theta$, $\cos 4\theta \approx 1 - \frac{16\theta^2}{2}$, $\tan\theta \approx \theta$

Sum $\approx 1 + 3\theta - \frac{16\theta^2}{2} + 2\theta$
$\approx 1 + 5\theta - 8\theta^2$
Answer: $p = 5$, $q = -8$

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7 Two independent binomials (All boards)

X ~ B(9, 0.4) and Y ~ B(12, 0.6) are independent.
Find P(X = 3 and Y ≤ 7), to 3 significant figures.

P(X = 3) = 0.2508
P(Y ≤ 7) = 0.5618
Independent, so multiply: 0.2508 × 0.5618 = 0.141
Answer: 0.141

8 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0 e^{kt}$, where t is in hours. Initially N = 200 and after 2 hours N = 300.
Find (a) k to 3 s.f., (b) when N reaches 1600, to 3 s.f.

$N_0 = 200$, so $300 = 200\,e^{2k}$, $e^{2k} = 1.5$

(a) $k = \frac{1}{2} \ln 1.5 = 0.2027$

(b) $200\,e^{kt} = 1600$, so $t = \frac{\ln 8}{k} = 10.26$ hours

Answer: (a) k = 0.203 (b) t = 10.3 hours

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that 12m + 15n = 35.

Assume there are integers m and n with 12m + 15n = 35.
12m + 15n = 3(4m + 5n).
So the left-hand side is a multiple of 3.
But 35 = 3 × 11 + 2 is not a multiple of 3.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 3, 35 is not)

10 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A marble is projected vertically upwards from ground level with speed 19.6 m s^{-1}.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top v = 0: $0 = 19.6^2 - 2 \times 9.8 \times H$
H = 384.16 ÷ 19.6 = 19.6
(b) s = 0: $0 = 19.6\,t - 4.9t^2$, so t = 19.6 ÷ 4.9
Answer: (a) 19.6 m (b) 4 s