Day 140 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial critical region (All boards)

$X \sim B(25, p)$. H_0: $p = 0.1$ is to be tested against H_1: $p > 0.1$ at the 10% significance level.

Find the critical region and the actual significance level.

Under H_0, $X \sim B(25, 0.1)$

$P(X \geq 5) = 0.0980 \leq 0.1$

$P(X \geq 4) = 0.2364 > 0.1$

The critical region is $X \geq 5$

Answer: $X \geq 5$; actual significance level 9.80%

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^3)^3 \times 4x}{4x^8}$.

$(4x^3)^3 = 64x^9$

Numerator $= 64 \times 4x^{10} = 256x^{10}$

Divide: $\frac{256}{4} = 64$ and $x^{10} \div x^8 = x^2$

Answer: $64x^2$

3 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 7 cm and arc length 4.9 cm.

Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{4.9}{7} = 0.7$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.7 = 17.15$

Answer: (a) 0.7 rad (b) 17.15 cm^2

4 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(7, 11) and B(2, 14), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$49 + (h - 11)^2 = 4 + (h - 14)^2$

Expanding and cancelling h^2: $-22h + 170 = -28h + 200$

$h = 5$

$r^2 = CA^2 = 85$

Answer: centre (0, 5), $x^2 + (y - 5)^2 = 85$

5 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 15°C: $\frac{dT}{dt} = -k(T - 15)$. Initially $T = 95$ and after 8 minutes $T = 45$.

Find k to 3 s.f. and the time to cool to 25°C, to 3 s.f.

$\frac{1}{T - 15}\,dT = -k\,dt$, so $\ln(T - 15) = -kt + c$, i.e. $T = 15 + 80e^{-kt}$

$t = 8$: $45 = 15 + 80e^{-8k}$, so $e^{-8k} = \frac{3}{8}$

$k = \frac{1}{8}\ln\frac{80}{30} = 0.1226$

$25 = 15 + 80e^{-kt}$: $t = \frac{1}{k}\ln\frac{80}{10} = 16.96$

Answer: $k = 0.123$, $t = 17.0$ minutes

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6 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 3x^2 + 6x - 2$ and the line $y = -6x - 2$ meet at the points A and B.
Find the coordinates of A and B.

$3x^2 + 6x - 2 = -6x - 2$
$3x(x + 4) = 0$
$x = -4$ or $x = 0$
$y = -6x - 2$: $y = 22$ and $y = -2$
Answer: A(−4, 22), B(0, −2)

7 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected horizontally at 196 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 48 s later, to 1 d.p. below the horizontal.

Vertical speed = $9.8 \times 48 = 470.4$
Speed = $\sqrt{(196^2 + 470.4^2)} = 509.6$
$\tan \alpha = 470.4 \div 196$, $\alpha = 67.4°$
Answer: 509.6 m s^{-1} at 67.4° below horizontal

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{64}$ for $0 \leq x \leq 8$, and 0 otherwise.

(a) Find P(X > 2).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{64}$

(a) $P(X > 2) = 1 - F(2) = \frac{15}{16}$

(b) $F(m) = 0.5$: $\frac{m^2}{64} = \frac{1}{2}$

$m = 5.66$

Answer: (a) $\frac{15}{16}$ (b) 5.66

9 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A model car moves in a straight line. At time t seconds its acceleration is $4t + 1$ m s^{-2}.
When t = 0 the velocity is −2 m s^{-1}. Find the velocity when t = 5.

$v = \int (4t + 1)\,dt = 2t^2 + t + c$
$t = 0$, $v = -2$ so $c = -2$
$t = 5$: $v = 53$
Answer: 53 m s^{-1}

10 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

to find the value of $\lim_{\theta \to 0} \frac{1 - \cos 3\theta}{\theta \sin 4\theta}$.

$1 - \cos 3\theta \approx 1 - (1 - \frac{3^2\theta^2}{2}) = \frac{9\theta^2}{2}$
$\theta \sin 4\theta \approx \theta \times 4\theta = 4\theta^2$
Ratio = $\frac{9\theta^2}{(2 \times 4\theta^2)} = \frac{9}{8}$
Answer: $\frac{9}{8}$