Day 14 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moves in a straight line with constant acceleration. Its initial speed is 15 m s^{-1}, its final speed is 27 m s^{-1} and it travels 84 m.
Find the time taken.

$s = \frac{1}{2}(u + v)t$
$84 = \frac{1}{2} \times (15 + 27) \times t$
$t = 168 \div 42$
Answer: 4 s

2 Binomial expected frequency (All boards)

Each of 11 people independently supports a team with probability 0.6; X is the number of supporters.
This is done 100 times. Estimate the number of times that X = 6.

$X \sim B(11, 0.6)$
$P(X = 6) = {}^{11}C_6 \times (0.6)^6 \times (0.4)^5 = 0.2207$
Expected $= 100 \times 0.2207 = 22.07$
Answer: about 22 times

3 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 3 N and 4 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 3 N force, to 1 d.p.

Magnitude $= \sqrt{(3^2 + 4^2)} = \sqrt{25}$
$\tan \alpha = 4 \div 3$
Answer: 5 N at 53.1°

4 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 12 cm and angle 300°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 300° = \frac{5\pi}{3}$ radians
Arc length $= r\theta = 20\pi$ cm
Area $= \frac{1}{2}r^2\theta = 120\pi$ cm^2
Answer: arc 20π cm, area 120π cm^2

5 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -x + 5$ for $-4 \le x < -2$.
Find the range of f.

f is decreasing, so the extreme values occur at the ends of the domain.
$f(-4) = 9$ and $f(-2) = 7$
$x = -4$ is included and $x = -2$ is excluded, so $f = 9$ is included and $f = 7$ is excluded.
Answer: $7 < f(x) \le 9$

6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = 2xy$, given that $y = 1$ when $x = 0$.
Give y in terms of x.

$\frac{1}{y}\,dy = 2x\,dx$, so $\ln y = \frac{2x^2}{2} + c$
$x = 0, y = 1$: $c = \ln 1$
$y = 1\,e^{1x^2}$
Answer: $y = e^{x^2}$

7 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a square number cannot be written in the form 7k + 6, where k is an integer.

Write the integer as n = 7q + j, where j = 0, 1, ..., 6.

n^2 leaves the same remainder as j^2 when divided by 7.

j = 0, 1, 2, 3, 4, 5, 6 give remainders 0, 1, 4, 2, 2, 4, 1.

The only possible remainders are 0, 1, 2, 4, never 6.

Answer: possible remainders 0, 1, 2, 4, never 6

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{4}$ for $0 \leq x \leq 2$, and 0 otherwise.

(a) Find P(X > 1).

(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{4}$

(a) $P(X > 1) = 1 - F(1) = \frac{3}{4}$

(b) $F(m) = 0.5$: $\frac{m^2}{4} = \frac{1}{2}$

m = 1.41

Answer: (a) $\frac{3}{4}$ (b) 1.41

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 5x - 14$ meets the circle $x^2 + (y + 1)^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = 5x - 14$: $x^2 + (5x - 13)^2 = 65$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$, so x = 1 or x = 4

$y = 5x - 14$: A = (1, −9), B = (4, 6)

$AB^2 = 3^2 + 15^2 = 234$

Answer: A(1, −9), B(4, 6), AB = $3\sqrt{26}$

10 Area under a curve (All boards)

The diagram shows the curve $y = x(4 - x)$ and the line $y = x$.
Find the exact area of the shaded region R.

Intersect: $x(4 - x) = x$, so x = 0 or x = 3

Area $= \int_0^3 (x(4 - x) - x)\,dx = \int_0^3 (3x - x^2)\,dx$

$= [\frac{3x^2}{2} - \frac{x^3}{3}]_0^3 = \frac{9}{2}$

Answer: $\frac{9}{2}$