Day 139 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Solving exponential equations using logarithms (All boards)

Solve $2^x = 3^{x+2}$, giving your answer to 3 s.f.

$x \ln 2 = (x + 2) \ln 3$

$x(\ln 2 - \ln 3) = 2 \ln 3$

$x = \frac{2 \ln 3}{\ln 2 - \ln 3} = -5.419$

Answer: $x = -5.42$

2 Poisson distribution (CIE)

$X \sim Po(5)$, modelling the number of emails received per hour.
Find, to 3 significant figures,
(a) $P(X \geq 4)$
(b) $P(X \leq 4)$

(a) $P(X \geq 4) = 1 - P(X \leq 3) = 0.735$

(b) $P(X \leq 4) = P(X = 0) + ... + P(X = 4) = 0.440$

Answer: (a) 0.735 (b) 0.440

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(0, -4)$ and $(2, -1)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= (\frac{0+2}{2}, \frac{-4+(-1)}{2}) = (1, -\frac{5}{2})$

(b) Gradient $= \frac{3}{2}$

$y + 4 = \frac{3}{2}x$

Multiply out and collect terms: $3x - 2y - 8 = 0$

Answer: (a) $(1, -\frac{5}{2})$ (b) $3x - 2y - 8 = 0$

4 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

7 values of x are coded using $y = \frac{(x - 100)}{2}$.

$\Sigma y = 7$ and $\Sigma y^2 = 177$.
Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 7 \div 7 = 1.000$

Mean of $x = 100 + 2 \times 1.000 = 102$

Variance of $y = \frac{177}{7} - (1.000)^2 = 24.29$

SD of $y = 4.928$, so SD of $x = 2 \times 4.928 = 9.86$

Answer: mean 102, standard deviation 9.86

5 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. A and B are the turning points. Find the turning points of:
(a) $y = f(x + 1)$ (b) $y = f(x) + 5$

(a) $y = f(x + 1)$: A$(-2, 2)$ maps to $(-3, 2)$, B$(0, -2)$ maps to $(-1, -2)$

(b) $y = f(x) + 5$: A$(-2, 2)$ maps to $(-2, 7)$, B$(0, -2)$ maps to $(0, 3)$

Answer: (a) $(-3, 2)$, $(-1, -2)$ (b) $(-2, 7)$, $(0, 3)$

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6 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 24t + 18$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $6(t - 1)(t - 3) = 0$, so $t = 1$ or $t = 3$

First time $t = 1$

$s = \int_0^1 v\,dt = [\,2t^3 - 12t^2 + 18t\,]$ from 0 to 1

Answer: t = 1 s; 8 m

7 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $12\sin^2 x + 3 - 13\sin x = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(4\sin x - 3)(3\sin x - 1) = 0$, so $\sin x = \frac{3}{4}$ or $\sin x = \frac{1}{3}$

$\sin x = \frac{3}{4}$: $x = 48.6°, 131.4°$

$\sin x = \frac{1}{3}$: $x = 19.5°, 160.5°$

Answer: x = 19.5°, 48.6°, 131.4°, 160.5°

8 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 7 and common ratio 3.
Find the smallest value of n for which the sum of the first n terms exceeds 20000.

$S_n = \frac{7(3^n - 1)}{2} > 20000$

$3^n > 5715.29$

$n > \frac{\lg 5715.29}{\lg 3} = 7.874$

So the smallest integer n is 8

Answer: n = 8

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-1 + 1\lambda = 0 + 2\mu$
j: $3 + (-2)\lambda = -1 + (-2)\mu$
Solving i and j: $\lambda = 3$, $\mu = 1$
k check: $4 + (-1)(3) = 1$ and $0 + 1(1) = 1$, equal

Point: $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{-3} \\ \mathbf{1} \end{pmatrix}$

10 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity ($\mathbf{i}$) m s^{-1} at time t = 0 and moves with constant acceleration ($4\mathbf{i} + 4\mathbf{j}$) m s^{-2}.
Find its velocity and speed when t = 5.

$v = u + at = (\mathbf{i}) + 5(4\mathbf{i} + 4\mathbf{j})$

$= 21\mathbf{i} + 20\mathbf{j}$

Speed = $\sqrt{(21^2 + 20^2)}$

Answer: $\mathbf{v} = (21\mathbf{i} + 20\mathbf{j})$ m s^{-1}; speed 29 m s^{-1}